

BSc test by course unit

MAS111 Convergence and Continuity

05 November 2007 11:00

Duration 40 minutes

You should attempt ALL questions. Marks awarded are shown next to the question.

Calculators are not permitted in this test. The unauthorised use of a calculator constitutes an examination offence.

**You are not allowed to start reading the question paper
until instructed to do so by the invigilator**

- 15 1. The following statements hold for all real numbers x, y

1. $x0 = 0$
2. $0 < x < y \iff 0 < \frac{1}{y} < \frac{1}{x}$.

Prove them directly from the axioms for \mathbb{R} . Indicate *precisely* which axioms are used at *each* stage of the proofs.

2. Limits

- 5 (a) What is the definition of a limit of a sequence a_n ?
- 10 (b) Prove that if a sequence a_n converges to a limit a then any subsequence of this sequence converges to the same limit.
- 20 (c) Prove that if $a_n = q^n$, where $1 > q > 0$ then $\lim_{n \rightarrow \infty} a_n = 0$ (use the property of limits stated in the previous problem).
- 20 3. Prove from the first principles that if $\lim_{n \rightarrow \infty} a_n = a$ and $\lim_{n \rightarrow \infty} b_n = b$, then $\lim_{n \rightarrow \infty} (a_n + b_n) = a + b$.
- 10 4. Use the basic properties of limits to compute

$$\lim_{n \rightarrow \infty} \frac{n^2}{3n^2 + 1} = \frac{1}{3}$$

- 20 5. Prove that

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$$

hence compute

$$\lim_{n \rightarrow \infty} \sqrt{n^a + 1} - \sqrt{n^a},$$

where $a \in \mathbb{N}$.

End of examination.