Research Talk for Paris Trip

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1 Introduction

My research focuses on analyzing the following *Langevin equation*:

$$\dot{X}_t = -\gamma X_t + \xi_t, \tag{1.1}$$

where we work on a special stochastic process $(\xi_t)_{t\geq 0}$, called *Non-Markovian* and *Non-Gaussian* (NMNG) noise, defined by;

$$\xi_t = \sum_{i=1}^{N_t} A_i h(t - T_i) \tag{1.2}$$

for *iid* random variables A_i and $T_i \sim U(0,t)$ and N_t forms the *Poisson process.* In practice the term T_i delays the *memory kernel* h and gives rise to memory effects to the NMNG noise ξ_t, hence the non-Markovianity.

Our specific aims are:

1. Analyzing the behaviour of the NMNG noise ξ_t.
2. Behavior of NMNG noise and X_t under different memory kernels h.
3. Finding the joint PDF of X_t and ξ_t. For the last point, we aim to generalize the *van Kampen equation*[1] governing the evolution of the joint PDF of X_t under Gaussian white noise, $\mathrm{d}W_t$. By using $\mathrm{d}W_t$ as the noise, the resulting solution X_t of $\dot{X}_t = -\gamma X_t + \sigma\,\mathrm{d}W_t$ forms the so-called *Ornstein-Uhlenbeck process.*
4. Finding the solution X_t.
5. Finding the characteristics of X_t such as its MSD $\langle X_t^2 \rangle$ or more generally the auto-correlation $\langle X_t X_{t+\tau} \rangle$.
6. Finding the long-term behavior of ξ_t. Here, we aim to show convergence in distribution of ξ_t to Lévy processes, in detail Compound Poisson process and Brownian motion.

For the duration of the talk I will discuss the method of finding the *characteristic functional* of ξ_t and therefore X_t and use that to get the above-mentioned aims.

I will try to make this talk as approachable and fun as possible :) but I suggest you, especially to those without any background in stochastic calculus, to have some sort of understanding of the above-mentioned *italized* points as well as *Ito's Lemma* and *Kolmogorov Forward Equation* (aka *Fokker-Plank Equation*). Other cool papers to read:

- Z. Physik B 31, 407-416 (1978) by P Hanngi (for generalized Langevin equations)
- J. Phys. A: Math Gen. 30, 8427-8444 (1997) by M Caceres (for more information on the Ornstein-Uhlenbeck process)
- Physical Review E 58:1, 919-924 (1998) by A Fulinski (for non-Markovianity application)

[1] If interested refer to p.5 of van Kampen's paper here: `http://www.sbfisica.org.br/bjp/files/v28_90.pdf`