Writing Mathematics at Advanced Level: Part II

Franco Vivaldi
School of Mathematical Sciences

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Title, abstract, and the beginning of the introduction set the tone for the rest of the document.

The opening paragraph

A thesis in group theory

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> *Let $GL(d, q)$ denote the general linear group of invertible $d \times d$-matrices over the finite field $\mathbb{F}_q$ with q elements. In computational group theory, it is of interest to calculate the order n of an element $A \in GL(d, q)$, i.e.,*

$$n = \min\{j \in \mathbb{N} : A^j = I\}$$

> *where I is the $d \times d$ identity matrix. The integer d can be large, e.g., $d > 100$.*

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- The last sentence hints to the type of problems to be addressed.

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- Formal definitions and symbols are given much later.

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• Writing for specialists, without apologies. The author relies on the reader's familiarity with half a dozen advanced concepts.

• In a single sentence, the author explains the meaning of the expression *ergodic optimisation* (the thesis' main topic), and introduces three important symbols. The notation is standard for the subject area.

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> Let K be an algebraic extension of $\mathbb{Q}_p$, and let $\mathcal{O}$ be its integer ring with maximal ideal $\mathcal{M}$, and residue field k. If $\overline{K}$ is an algebraic closure of K, we denote by $\overline{\mathcal{O}}$ and $\overline{\mathcal{M}}$ the integral closure of $\mathcal{O}$ in K and the maximal ideal of $\overline{\mathcal{O}}$, respectively.

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One cannot proceed without the cited publication (by the same author); most readers will stop reading here.

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The subject area may seem narrow, but in fact it is still broad; a thesis with this title would suggest a literature survey. A thesis with an original research component would require a more specific title.

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A rare instance of a research problem with a non-technical description, which makes a good title. The preposition 'on' lends an authoritative tone to it, promising some general result.

Toward a theory of Pollard's ρ-method [Eric Bach, *J. Information and Computation* **90** (1991) 139–155.]

A very eloquent title, which suggests that some progress has been made on a topic not yet supported by a theory. This would be an ideal title for a thesis. The use of the symbol ρ is justified, because the aforementioned method is invariably associated to it.

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The two terms belong to two different areas of mathematics, and their unexpected juxtaposition suggests interdisciplinarity. A short and effective title, perfect for a thesis.

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An excellent thesis title, enlivened by a colon. After announcing the research area, a clever reference to an epic Western hints to the specific topic (badly approximable numbers).

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Another provocative title-sentence —primality testing is notoriously difficult— which requires a fair deal of self-confidence. [This paper won the author the Chauvenet prize, awarded by the Mathematical Association of America to the author of an outstanding expository article on a mathematical topic.]

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- The abbreviation 'sup', for 'supremum' is meant for formulae, not text (much like 'lim', for limit).

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• The second sentence is of the form '*an example of this is that*'. The emphasis is placed on the word '*example*', yet the term '*Frenkel-Korontova model*' surely is more significant.

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IMPROVED ABSTRACT. *We investigate properties of minimal solutions of discrete periodic variational problems, in the multi-dimensional case. These generalise one-dimensional problems, such as the well-known Frenkel-Korontova model. We select a family* ...

Example 3

ABSTRACT. *The logistic map is a well-studied map of the unit interval into itself. However, if we treat x as a discrete variable, as is done in any computer, then every orbit is eventually periodic. Thus the aperiodic behaviour that the continuous map displays for some value of the parameter r cannot be obtained from computer simulation. We investigated* ...

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• There are many general terms (map, unit interval, discrete variable, eventually periodic, etc.) to help us discern the subject matter.

The second and third sentences motivate the study.

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• The symbol r suffers from similar problems. We are told it's a parameter; we guess that it parametrises the logistic map.

IMPROVED ABSTRACT. *We consider the logistic map, a well-studied one-parameter family of maps of the unit interval. If the domain of this map is discretised, as happens in any computer simulation, then, necessarily, all orbits become eventually periodic. Thus the aperiodic orbits observed for certain parameter values no longer exist. . . .*

Example 3, ctd

We examine the second part of the previous abstract.

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• What does '*follow*' mean? Presumably it indicates some form of
convergence.

We sharpen up these vague statements.

IMPROVED ABSTRACT. ... *We investigate differences and similarities between the original map and its discrete approximations. We* **provide evidence** *that the limit cycles of the discrete map converge to the unstable orbits of the original map,* **in a sense to be made precise.**

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The last expression states that the thesis contains experimental data or heuristic arguments that support a clearly formulated notion of convergence. (We hope that this is the case in the present thesis!)

Abstract 4

ABSTRACT. *Let* $f : X \to X, X = [0,1)$, *be an IET (interval exchange transformation) ergodic with respect to the Lebesgue measure on* X. *Let* $f_t : X_t \to X_t$ *be the IET obtained by inducing* f *to* $X_t = [0,t)$, $0 < t < 1$. *We show that*

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- f is assumed to have a certain property ("*ergodic . . . on X*").

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ORIGINAL ABSTRACT

Let $f : X \to X, X = [0, 1)$, be an IET (interval exchange transformation) ergodic with respect to the Lebesgue measure on X. Let $f_t : X_t \to X_t$ be the IET obtained by inducing f to $X_t = [0, t)$, $0 < t < 1$. We show that $X_{wm} = \{0 < t < 1 : f_t$ is weakly mixing$\}$ is a residual subset of X of full Lebesgue measure. The result is proved by establishing a Diophantine sufficient condition on t for f_t to be weakly mixing.

IMPROVED ABSTRACT

Let f be an interval exchange transformation (IET) of the unit interval, ergodic with respect to the Lebesgue measure, and let f_t be the IET obtained by inducing f on the sub-interval $[0, t)$, with $0 < t < 1$. We show that the set of values of t for which f_t is weakly mixing is a residual subset of full Lebesgue measure. The result is proved by establishing a Diophantine condition on t, which is sufficient for weak mixing.

Exercises

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ABSTRACT. *In this project the author examines some properties of continued fractions (CF). In the beginning, definitions, notations, and basic results and theorems are shown. Periodic continued fractions and best approximations are examined subsequently in depth. We examined a number of applications to mathematics and astronomy.*

Abstract. *The most representative example of a 2-dimensional area-preserving twist map is the standard map, which is studied. Orbits for which the momentum p grows linearly (plus a periodic function) are shown to exist, classified and determined numerically. These orbits are the accelerator modes. The linear stability of these orbits is determined. The range in the parameter values for which they exist is also determined.*

ABSTRACT. *We consider equal parameter generalized quadrangles, GQ(s, s). All GQs of order 2 and 3, GQ(2, 2) and GQ(3, 3), are known. It is conjectured all the GQ of order five are known as well. The known GQ of order 5 is the symplectic GQ, W(5). W(5) along with its dual are conjectured to be the only GQs of order 5. The construction of a symplectic GQ is given and then used to construct the known GQ of order 5. Information about GQ was gathered, including some basic combinatorics, affine GQ, and incidence matrices in an attempt to prove the above conjecture.*

ABSTRACT. *Under the boundary condition on the initial value $T_k(0)$ ($T_k(0) > 0$; $k = 1, 2, \ldots$) that $T_k(0) \to 0$ ($k \to \infty$), we integrate the semi-infinite system of non-linear differential equations $\dot{T}_k = 2T_k(T_{k+1} - T_{k-1})$, ($k = 1, 2, \ldots$; $T_0 = 0$) to obtain their general solution. We further investigate the asymptotic time behaviours of this general solution as $t \to \pm\infty$.*

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