# Mathematical & Scientific Writing

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**Abstract**

A short course on mathematical writing for beginning PhD students in Mathematics, Physics and Engineering.

# Contents

# 1 Mathematical & Scientific Writing: An Introduction

## 1.1 Introducing some people

This postgraduate training course has been developed by Will Clavering and Franco Vivaldi in the School of Mathematical Sciences, and Sally Mitchell from the Language and Learning Unit. As well as developing students' own scientific writing, the course also prepares them to act as teaching assistants on Prof. Vivaldi's undergraduate Mathematical Writing module. Some of the examples and exercises in this course are drawn from the undergraduate material, available as a web-book. When discussing material from the undergraduate course, we make the distinction between postgraduate students (attending this course) and undergraduate students (on Prof. Vivaldi's undergraduate course).

This year (Autumn 2007), the course was delivered by Will Clavering with assistance from Sally Mitchell and Giles Martin. Sally works in the Language and Learning Unit which, amongst other inter-disciplinary projects, offers one-to-one writing and study skills tutorials. Giles Martin is a former Mathematics researcher who now works in Educational and Staff Development (ESD), which provides training courses for College staff and research students.

## 1.2 Objectives of the course

The advertised objectives state that, by the end of the course, participants should be able to:

- Combine style and precision when communicating mathematical ideas;
- Provide feedback and assessment for written work;
- Write an effective scientific abstract and research summary.

Some other objectives:

- Help improve the writing of research students;
- Provide some useful reference material;
- Provide training for students wishing to assist with the undergraduate course;
- Improve the course itself.

## 1.3 Overview of Vivaldi's undergraduate course

The undergraduate course, associated with this course, is a second-year module in the School of Mathematical Sciences at Queen Mary. As well as covering mathematical definitions, proofs, and some elementary logic in the syllabus, the course draws on a broad range of first-year mathematics. Coursework contains a high proportion of written exercises in which the students are encouraged to engage more deeply with the material, as well as to develop their writing.

## 1.4 Overview of this course

Scientific research students are expected to complete a wide range of written assignments during the course of their study. An illustrative list of these might be as follows: research papers (solo and co-authored); seminar slides; Ph.D. thesis; conference posters; lab reports; personal notes (over the course of 3/4 years); programming and LaTeX annotation; personal communication; undergraduate script feedback; research proposals; periodical research reports. The ESD catalogue and study skills section of the Library provide a number of courses and books addressing many of these tasks in detail. These resources are particularly useful for when planning and re-drafting your assignment; they typically involve working back from a model structure to improve your own work. Such a top-down approach, however, does not account for the considerable overlap between these tasks – the ingredients for writing a good paper are similar to those required for a good seminar. An objective approach to your work, appreciation of your audience and clarity in expressing mathematical ideas are all important when explaining science, through any medium. In an attempt to categorise every possible assignment that a research student might undertake, the important messages can be lost.

In this course, we concentrate on the universal tools of mathematical writing. We begin, in Section 2, by considering how to define and describe mathematical objects and how language can help us with this process. In Section 3 we examine the structure of mathematical arguments (particularly proofs) and discuss the importance of the writer's audience. We illustrate how to improve writing with comments and annotation, clear typesetting, emphasis, and by following some simple rules. In Section 4 we study some exercises from the undergraduate course, from which we can draw parallels with our own work.

In Section 5 we consider the audience for a research paper and how to structure our writing for this audience. This approach highlights the importance of introductory and concluding sections, and some suggestions for an effective title – considerations that apply to most other forms of scientific writing.

The course can be delivered in three sessions of two hours each. The notes and examples below form the basis for Session 1 and much of Session 2. At the end of the second session, and the beginning of the third, we discuss a number of expository papers, to illustrate topics from the course. Some notes on what was said this year are provided in Section 8. During the week-long break between the second and third sessions, participants are asked to prepare a short piece of writing, as outlined in section 9. Session 3 is almost entirely devoted to a workshop based on these assignments.

During the course we shall distribute some handouts, based on material in the appendices and references contained in these notes. We also look through some of the writing resources listed in Section 7.

# 2 Building Blocks

We begin by taking a highly atomistic approach to writing. In this section we focus on short descriptions, definitions, and abstractions of mathematical objects. We then discuss how sentence structure, punctuation and emphasis can be used effectively when writing about these objects.

## 2.1 Turning words into symbols

In this example, undergraduate students are asked to rewrite a mathematical object without using symbols;

| |
|---|
| $\{x \in \mathbb{R} : 0 < x < 1\}$. |

Here we might be happy with a direct interpretation,

| |
|---|
| The set of real numbers *strictly* between 0 and 1. |

Since this set is a familiar object, however, we can improve the description for our reader by identifying it:

| |
|---|
| The open unit interval. |

This example emphasises the benefit of characterising expressions in terms of familiar objects.

Now we have another example (again a set),

| |
|---|
| **Exercise 1**<br><br>Rewrite the following set without using symbols:<br><br>$\{f : \mathbb{R} \to \mathbb{R} : \forall x \in \mathbb{Q},\ f(x) \neq 0 \Leftarrow x \neq 0\}$. |

Here a symbol-by-symbol interpretation (referred to as *robotic* in the undergraduate course notes) shows a lack of engagement with the mathematics.

> The set of functions with real domain and co-domain such that, for all rational numbers, the function is non-zero when its argument is nonzero.

By studying the mathematical object (in particular the contrapositive of the final condition) we can achieve something more readable. Our attention is also drawn to the key features of the set. A model solution from the course reads,

> **Intermediate working**
>
> $\{f : \mathbb{R} \to \mathbb{R} \,:\, \forall x \in \mathbb{Q},\, f(x) = 0 \Rightarrow x = 0\}$
>
> **Solution**
>
> The set of real functions that do not vanish at the rationals, except possibly at zero.

## 2.2 Levels of description & abstraction

In the next exercise, undergraduate students are required to provide two levels of description. Firstly they are asked for a *coarse* description, identifying the class to which the object belongs (set, function, polynomial, etc.). Secondly they must provide a *fine* description, which defines the object in question or characterises its structure.

> **Examples**
>
> $(x + y)^2 = x^2 + 2xy + y^2$
>
> - An algebraic identity.
> - An algebraic identity illustrating the expansion of the square of a binomial.
>
> $\sqrt{2 + \sqrt{3 + \sqrt{5}}}$
>
> - An arithmetical expression.
> - An arithmetical expression, with nested square roots.

In the above examples we should note the the distinction between expressions, identities, and equations.

Abstraction is useful when writing and speaking about mathematics, particularly when communicating with peers. If you have a complicated expression which you're absolutely compelled to put in a talk, then try to give a simple abstraction as well; your audience will appreciate it.

As a trivial exercise consider:

**Exercise 2**

Write a *coarse* and *fine* description of this object

$\frac{dy^2}{dx^2} - 3\frac{dy}{dx} - 2 = 0$

**Solution**

- A differential equation.
- A linear second-order (homogeneous) ordinary differential equation.

The fine description, in this case, gives us a great deal of information about the object – in particular an idea of what the solution might look like.

## 2.3 Graphical information

For the exercise in this section, the postgraduates students are split into small groups and asked to come up with a short description of a function that is plotted for them on a graph. A member of another group is then asked to reproduce this graph in front of the group, and we discuss the merits of the description. The examples used in the course are included at the end of these notes.

**Exercise 3**

In groups, agree on a short description of the function (one graph handed out to each group). One of your group will then read out your solution and a member of another group will attempt to re-draw the graph from your definition.

While some of the communication problems can be solved by simply recalling the correct technical language (odd/even functions, etc.), the postgraduate students have also found that comparisons with familiar graphical features are useful – a good description, for the irregular periodic function, mentioned that its graph looked like that of a Fourier transform.

## 2.4 Emphasis

We now discuss how to apply these abstract descriptions to our writing. Consider,

$\sin(\theta_1 - \theta_2) = \sin(\theta_1)\cos(\theta_2) - \cos(\theta_1)\sin(\theta_2)$,

where a two-level description might be written,

- A trigonometric identity.
- The formula for the sine of the difference of two angles.

**Exercise 4**

Consider the expression,

$\sin(\theta_1 - \theta_2) = \sin(\theta_1)\cos(\theta_2) - \cos(\theta_1)\sin(\theta_2)$.

Here are three possible ways that we could use the description to write about the object:

- This is a trigonometric identity. It is the formula for the sine of the difference of two angles.
- This is a trigonometric identity, the formula for the sine of the difference of two angles.
- This is a trigonometric identity, which gives us the formula for the sine of the difference of two angles.

Discuss the flow, emphasis and impact of each.

Using two sentences probably gives the statement most impact. The coarse description, however, may not merit this emphasis. Too many short sentences can also disrupt the flow of an argument.

The comma, in the second sentence, allows the statements to flow more easily. To see this, imagine how the next sentence might sound.

The final sentence emphasises the consequential sequence of the two statements.

Which of these is preferable will depend on what you're trying to emphasise, and the surrounding text.

We now pause to mention a common writing query, raised by the last example, which can be addressed in terms of mathematical definitions.

**Which and That**
The general grammatical rule is to use 'which' only when it is preceded by a comma or by a preposition, or when it is used interrogatively.

A mathematician might prefer the following explanation. If the object is already defined and you are supplying additional information, use '... , which'. On the other hand, if the secondary information is required in the definition, use 'that'.

**Examples**
This is my pen, which I brought here from home.

Where is the pen that I brought from home?

## 2.5 Tips for effective use of emphasis

We now consider a number of tips on providing emphasis to mathematical statements. Note that these are not universal rules, and it is important to avoid over-emphasising unimportant aspects of your work.

We now study some examples of bad practice and discuss how we might improve each.

**Use active voice.**
The set of real numbers is considered. **X**

This statement is weak and might well leave the reader asking, 'considered by who?' Such indefinite statements make the sentence difficult to read, or even misleading. Two improved efforts might read as follows (note, they both answer the implicit question).

We consider the set of real numbers.✓
Consider the set of real numbers.✓

**Place emphatic words at the end.**
The real numbers are not algebraically closed, even if they do form a field.**X**

Here the key statement is that the real numbers are not algebraically closed, with a supplementary remark, that they form a field. We can emphasise this more clearly by rearranging:

Even though they form a field, the real numbers are not algebraically closed.✓

**Say important things twice.**
To emphasise a key point, such as a subtle definition, think of two different ways to write it.

In the example earlier, we could have done the following.

$\{x \in \mathbb{R} : 0 < x < 1\}$.
This is the open unit interval, the set of real numbers strictly between 0 and 1.

Again, we stress that not all arguments need emphasising in this way. You should think about your audience – a second year undergraduate might like to be reminded of the properties of the open unit interval, whereas a postgraduate journal reader would probably not.

# 3 Proofs: Titles, Structure and Keeping the Reader Informed

We consider the following proof of a simple result from number theory. The mathematics is essentially correct; a reader, who is familiar with the proof, should recognise the key mathematical arguments. As a piece of mathematical writing, however, it is abominable.

**Exercise 5**

Study the following proof, then suggest how it might be improved for the reader.

**Proof:**

$$\begin{aligned} 2 \nmid n \Rightarrow \exists k \in \mathbb{Z}\ n = 2k+1. \\ n^2 - 1 = 4k(k+1). \\ \forall k \in \mathbb{Z}\ 2 \mid k(k+1). \ \square \end{aligned}$$

We have therefore proved the following:
**Theorem:** $\forall n \in \mathbb{Z}, 2 \nmid n \Rightarrow 8 \mid n^2 - 1$.

This kind of 'bare bones' mathematics is useful for rough working or taking lecture notes, where writing speed is paramount. In such cases the work is not intended to be read by anyone else.

Here is a list of possible improvements that a group of postgraduate students suggested:

- State first, prove second;
- Provide a title for the proof;
- Use self contained or well referenced notation;
- Add structural comments and informative annotation;
- Emphasise the result.

We now reflect on how each of these can be seen in the model solution from the undergraduate course:

**Theorem:** The square of an odd integer is equal to 1 plus a multiple of 8.

**Proof.** Let $n$ be a given odd integer. We have to show that there exists an integer $j$ such that $n^2 - 1 = 8j$.

Since $n$ is odd, there is an integer $k$ such that $n = 2k + 1$. We find

$$n^2 - 1 = (2k+1)^2 - 1 = 4k^2 + 4k + 1 - 1 = 4k(k+1) \qquad (1)$$

which shows that $n^2 - 1$ is divisible by 4. It remains to prove that the product $k(k+1)$ of two consecutive integers is even. There are two cases: if $k$ is even, we have finished; if $k$ is odd, then $k+1$ is even. In either case, our product is divisible by 2, and therefore there is an integer $j$ such that $k(k+1) = 2j$.

Inserting this expression in (1), we find

$$n^2 - 1 = 8j$$

as desired. $\square$

## 3.1 A brief note on punctuation

This section provides a list of some common forms of punctuation found in scientific literature. The remarks here are by no means a comprehensive guide to usage; this is more of a crash course. During this part of the course, the postgraduates were asked to study examples in the handout (Appendix 4 of the Oxford Advanced Learner's Dictionary). Examples of punctuation in scientific writing can also be found in the handouts for this course, and postgraduate students might like to discuss these during the course.

**Full stop**
The simplest way to end a statement. Used also to denote abbreviation.

**Comma**
Used to punctuate lists, separate clauses, enclose side remarks, and begin quotations. Commas should not be used between two independent clauses that form sentences on their own.

Commas have a number of uses in punctuation. You can avoid confusing commas in lists with those used for other purposes (or even secondary lists), by breaking up sentences or using semicolons.

**Colon**
Used to deliver a promised statement, introduce a list, or begin a longer quotation.

The colon is used sparingly in most forms of literature. In scientific literature, however, it is more frequent; we often wish to define an object in words, then deliver a symbolic description. An example in the next section illustrates this point.

**Semicolon**
May be used as a stronger version of a comma, especially to avoid ambiguity in lists. It can also be used as a weaker version of a full stop: to join two related, but independent, clauses.

**Brackets**
Used to separate information, or a comment, from the rest of a sentence. The surrounding text should read correctly with the bracketed text removed.

**Dash**
May be used to to enclose a side remark, or instead of a semicolon or colon to introduce a remark. Note that in LaTeX the dash is input as two or three hyphen/minus signs – so it looks like this.

When dashes are used to enclose a remark, they shift emphasis to the remark itself. You might like to read aloud such constructions in a different, more dramatic, voice. As with the other forms of emphasis, you should not overdo this construction in your writing. You should also be careful using hyphens around numbers and symbols; they are easily confused with the minus sign.

## 3.2 Introducing mathematics

Building on Exercise 5, we now discuss introducing mathematical expressions to our written text. Important expressions and results (theorems, lemmas, etc.) usually need to be displayed on their own. Punctuation is important here to ensure that the work reads correctly – your reader may want to skim over some of the more complex expressions initially.

In terms of style, these mathematical objects are similar to long quotations.

Here we highlight some simple examples of good practice.

We now prove the following elementary result.
**Theorem:** The square of an odd integer is equal to 1 plus a multiple of 8.

... , which leads to an elementary result:
**Theorem:** The square of an odd integer is equal to 1 plus a multiple of 8.

For mathematical expressions, the general rule is to maintain sentence structure. It might help to read the work over and replace each equation with a word like 'something', 'whatever', or just a grunt; in order to check the flow.

**Bad:**
Inserting this expression in (1):

$$n^2 - 1 = 8j$$

which is what we were required to prove.

**Good:**
Inserting this expression in (1), we find

$$n^2 - 1 = 8j$$

as desired.

## 3.3 Rules about symbols

Although there may be a few exceptions, these rules can be applied in almost all situations.

**Separate symbolic expressions with words**
Bad: Now consider $f(x)$, $x < 0$.
Good: Now consider $f(x)$, where $x < 0$.

**Don't start sentences with symbols**
Bad: This number is clearly even. $4n + 3$, on the other hand, is odd.
Good: This number is clearly even. On the other hand, $4n + 3$ is odd.

**Replace the following symbols with words**
Bad: $\therefore$, $\Rightarrow$, $\forall$, $\exists$.
Good: therefore, implies, for all, there exists.

The symbols should, of course, remain in logical statements such as set definitions, but should not be used as shorthand in a proof.

**Be careful introducing subscripts and superscripts**
Double indexed expressions such as $x_{\alpha_i}$ can often be avoided by reconsidering your original definitions.

**Don't use the same notation for different things**
This sounds obvious, but is ignored more often than you might think. Common symbols (such as $f(x)$, $n$, $k$, $A$ etc.) are the most likely candidates.

**Don't introduce symbols unless you intend to use them**
Again, this sounds obvious, but statements like
'A differentiable function $f$ is continuous.' **X**
are very common, and regularly lead to using the same notation for different things.

# 4 Assessment & Feedback

In this section, we consider some examples of undergraduate solutions to written exercises. This will prepare the postgraduate students for marking written assignments, interpreting and devising marks schemes, and providing feedback in exercise classes. The principles of providing effective feedback also apply to peer review, something we explore in the third session of the course. They also apply to editing manuscripts, co-authoring papers, and writing personal correspondence on scientific matters.

## 4.1 Overview of marking mathematics

Marking mathematical scripts is often a fairly straightforward process. The marks available will have been largely divided by the course tutor amongst a number of short exercises; although the marker may have to use their discretion in awarding partial credit. It's then simply a matter of identifying and penalising errors (although providing useful feedback can be more of a challenge).

## 4.2 Single sentence assignments

Even for very short written exercises, we cannot easily adapt the usual marking model. Consider an example in which the undergraduate students are asked to convert symbols to words.

$\{x \in \mathbb{Q} : x \notin \mathbb{Z}, 1/x \in \mathbb{Z}\}$. [3 marks]

Here, the following answer deserves full marks.

- The set of proper fractions with unit numerator.

Note that the definition gives reciprocals of integers, excluding 0 (since $1/0 \notin \mathbb{Q}$) and 1 (since $1/1 = 1 \in \mathbb{Z}$).

**Exercise 6**

Assess the merits of the following three descriptions of the set,

$\{x \in \mathbb{Q} : x \notin \mathbb{Z}, 1/x \in \mathbb{Z}\}$. [3 marks]

1. The set of reciprocals of rational numbers excluding zero and one.
2. The set of rationals such that the number itself is not an integer but its reciprocal is.
3. The set of reciprocals of integers so long as the integer is not zero and the integer does not have modulus one.

The first answer is plain wrong: it does not give a correct mathematical description of the object. This was marked **[0/3]**.

The second is technically correct but it's a symbol-by-symbol translation of the kind mentioned earlier. The undergraduate students are warned explicitly about this kind of answer, which allows them to avoid thinking about the properties and elements of the set. This answer was marked **[1/3]** – **robotic**.

The third example is again technically correct, although rather awkward to read (it's actually easier to understand if you read the second half of the sentence first). Here we would probably give a mark **[2/3]** – **clumsy**.

We also deduct marks for more basic writing mistakes, for which we use the following shorthand:

| | |
|---|---|
| **CAP** | – missing / incorrect use of capital letter |
| **FS** | – missing full stop |
| **S** | – spelling mistake |
| **S/P** | – singular verb with plural noun |
| **G** | – other grammatical error |
| $\notin$ | – use of symbols when prohibited |

## 4.3 Longer pieces: MicroEssay examples

In the exercise above, the marker's primary aim was to assess the correctness of each answer. The allocated marks and shorthand comments provide sufficient feedback for the undergraduate students to review their coursework, and relate their performance to the model solutions. For longer pieces of writing, more is required than simply a sequence of individually correct sentences. In order to provide representative marks, and useful feedback, we need to account for the structure and overall content of the piece. The following exercise is taken from the undergraduate course.

**Write a short essay on complex numbers** ($\notin$, 100 words).

Recall that $\notin$ means 'no symbols'. Other similar exercises require undergraduate students to write about differential equations, functions, sets, etc. We consider four solutions to this problem, and assign a preliminary grade to each.

**Exercise 7**

Provide a mark out of ten and some brief feedback for this essay (pass mark of 4/10 and first class boundary 7/10.)

**MicroEssay A**

The complex numbers form a field, under the usual addition and multiplication operations. Since they correspond to ordered pairs of real numbers, they can be represented by points in the plane. This is called the Argand diagram. Unlike the real numbers, the set of complex numbers is algebraically closed. Complex analysis, the study of complex functions, has produced a number of interesting results such as the Cauchy-Riemann equations and fractal sets such as the Mandelbrot set which you may have seen pictures of.

**Exercise 8**

Provide a mark out of ten and some brief feedback for this essay (pass mark of 4/10 and first class boundary 7/10.)

**MicroEssay B**

In mathematics, a complex number is a number of the form

$$a + bi$$

where $a$ and $b$ are real numbers, and $i$ is the imaginary unit, with the property $i^2 = -1$. The real number $a$ is called the *real part* of the complex number, and the real number $b$ is the *imaginary part*. Complex numbers can be added, subtracted, multiplied, and divided like real numbers and have other elegant properties. In some fields (in particular, electrical engineering, where $i$ is a symbol for current), the imaginary unit $i$ is instead written as $j$, so complex numbers are sometimes written as $a + jb$.

**Exercise 9**

Provide a mark out of ten and some brief feedback for this essay (pass mark of 4/10 and first class boundary 7/10.)

**MicroEssay C**

When solving a quadratic equation, it may happen that the discriminant be negative. If we still require that the equation has solutions, we are led to consider a quantity whose square is equal to $-1$, called the *imaginary unit.* Now, take two real numbers, multiply one of them by the imaginary unit, and add them together: it can be shown that any solution of a quadratic equation can be written in this way. The set of these expressions is called the set of complex numbers. It's an infinite set, equipped with the four arithmetical operations.

**Exercise 10**

Provide a mark out of ten and some brief feedback for this essay (pass mark of 4/10 and first class boundary 7/10.)

**MicroEssay D**

Complex numbers are made out of two real numbers and the imaginary unit. The first real number is called the real part of the complex number and the second is the imaginary part. Add the real part to the imaginary part times the imaginary unit to give you the complex number. Mathematicians usually call the imaginary unit $i$ (engineers often call it $j$) so that a complex number looks like $z = a + ib$. The number can also be written in polar form using as the modulus times the exponent of the imaginary number times the angle.

At this point we canvass the opinions of the class and look at the overall ratings of each piece in the following table.

| Essay | Fail 0-3 | Bare Pass 4 | 2nd Class 5-6 | 1st Class 7-10 |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |
| D | | | | |

So far, as markers, we have been provided with very little information about the aims of the task and marking criteria. As one might expect, this leads to rather a large spread of marks for most of the essays. We now look at the guidance provided to the markers of the undergraduate course, and consider the importance of assessment and feedback for the undergraduate students.

## 4.4 The distinction between assessment & feedback

When providing feedback on undergraduate scripts it is very tempting to view the feedback as simply a justification for our assessment. Whereas the assessment is primarily a summative judgement of the work's merit, we should remember that the aim of providing feedback is to aid the undergraduate student's learning.

Some advice from Queen Mary's code of practice:

**Key factors of good assessment:**

- Consistent across the class, and with other assignments;
- Indicative of the standard of work submitted;
- Succeeds in measuring what the exercise was designed to test.

**Key factors of good feedback:**

- Aids the undergraduate student's future learning;
- Indicates any particularly good/bad aspects of the work;
- Relates their work to the exercise objectives.

## 4.5 Developing a marks scheme

Essay C was in fact given as the model solution to this question. We now discuss how to create a marks scheme from this example.

**MicroEssay C**
When solving a quadratic equation, it may happen that the discriminant be negative. If we still require that the equation has solutions, we are led to consider a quantity whose square is equal to $-1$, called the *imaginary unit.* Now, take two real numbers, multiply one of them by the imaginary unit, and add them together: it can be shown that any solution of a quadratic equation can be written in this way. The set of these expressions is called the set of complex numbers. It's an infinite set, equipped with the four arithmetical operations.

The class is now invited to discuss the content and style of the model essay, to use as a guide for the mark scheme. A typical evaluation might be as follows:

- Content:
  - motivation / context;
  - definition;
  - three or four related mathematical statements.
- Style:
  - uncomplicated but correct language;
  - no symbols;
  - within word limit.

Using this evaluation, we discuss the grade boundary specifications for the undergraduate course.

**First class :** Must contain a clear and precise definition and two or three relevant, and connected, mathematical statements.
**Bare Pass:** Must contain either a correct definition or at least two correct, and relevant, mathematical statements.

Additional correct (incorrect) information may gain (lose) credit at the marker's discretion. A nice way of indicating good statements to the undergraduates is to mark them with a ✓. We also use **X** to indicate mathematical or logical errors (rather than linguistic errors for which we have other symbols).

In order to deal with the style criteria we really need some further guidance from the course tutor. For this essay the advice is as follows.

**Symbols & Plagiarism:** The undergraduate students were clearly warned about these issues in class. Numbers, and the four arithmetical operations, are allowed. Any statement containing other symbols or plagiarism (state the source) should be discounted.

**Word limit:** Don't enforce this too strictly. In the past, when undergraduate students have gone drastically over limit there is usually some other problem that's caused this. For an otherwise good essay that's gone significantly over limit (say 200 words) just knock off a single mark.

**Spelling, grammar, word order:** Indicate these errors but mark positively – so give a poorly written, but technically correct, statement partial credit.

This type of specification is necessary, as otherwise we might decide to cut off the essays at 100 words, or deduct points (negative marking) for each spelling mistake, grammatical error, etc.

## 4.6 Analysis and marking of the essays

Using the full marking scheme, we now reconsider our original thoughts about the three other scripts and enter new marks in the table below.

| Essay | Fail<br>0–3 | Bare Pass<br>4 | 2nd Class<br>5–6 | 1st Class<br>7–10 |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |
| D | | | | |

We find that, although there are still differences in opinion, this information moves the markers towards a close agreement on the numerical grades, and even more so on which grade range each script should achieve. We next move on to discuss the analysis, marking, and the actual feedback from last year's undergraduate course marker on the remaining scripts.

**MicroEssay A**

**Analysis**
This essay is fairly well written and it contains a number of well stated pieces of mathematical information. There is no definition, however, and the statements don't relate directly to one another; the piece is more of a list than a paragraph. The script is probably worthy of a second class grade. Here the feedback is very important as the undergraduate student appears to have struggled with the structure of the essay, and knowing what kind of statements to include, rather than with understanding or articulating the mathematics.

**Mark:** 6/10.
**Overall feedback:** Some good statements, but where is the definition?

**MicroEssay B**

**Analysis**
Although this essay contains correct mathematics, and a reasonable structure, there is practically nothing that we can award credit for. There is only one statement without prohibited symbols. There is even a discussion of terminology in the last sentence. The phrase 'elegant properties' is meaningless without any justification (this is the sort of thing that comes up frequently and may merit some new shorthand – perhaps $ for salesman language). All of these problems, however, pail into insignificance when we discover that the whole thing has been copied verbatim from Wikipedia!

**Mark:** 0/10.
**Feedback:** Plagiarised (Wikipedia);
Lots of $\notin$ symbols to get the point across.

Essay B raises a few interesting points. The plagiarism one is simple; for such short assignments undergraduate students won't do much beyond copying one another's papers (easy to spot if we parallel mark) and google-ing the essay title (which we can check ourselves). We also find that copying straight from Wikipedia articles is not going to get the undergraduate student very far (even if they don't get caught).

Although there is not much that can be done at the marking stage, if an undergraduate student brings MicroEssay B along to an exercise class then it could certainly be used to begin constructing an essay .

Here are some questions we might ask a student in an exercise class, to help them develop a suitable essay from MicroEssay B:

- Why are mathematicians interested in complex numbers?
- Can you re-write the definition without using symbols? (related to earlier coursework)
- Can you pick out the mathematical results from this article?
- What other results do you know involving complex numbers?

**MicroEssay D**

**Analysis**
This essay contains a rather poorly worded definition; in fact it is not clear that what is written contains a definition at all. The statements are all concerned, one way or another, with notation or symbols; there are also a few actual symbols. The content would be worthy of a bare pass, if it were well written, but as it is we should deduct a mark or two.

**Mark:** 3/10.
**Feedback:** Clumsy definition.
Robotic (last sentence).
Avoid discussing notation – see course notes and solutions for appropriate level of mathematical content.
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# 5 Some thoughts on Research Writing

## 5.1 How we choose papers to read

One of the hardest tasks in writing a scientific paper is developing a sense of objectivity to the work. You will probably have spent many months developing the scientific method of the paper but this is, in fact, the part least likely to be read by a member of your audience. Think about your own approach to reading scientific papers. Typically you might start with some kind of list containing the paper titles and names of the contributing authors, perhaps the contents page of a journal, an online feed, or email notification.

Let's say 10% of these papers sound interesting and relevant to your field of interest – then you've just rejected 90% based on the title alone. We'll leave those unlucky 90% for now and focus on the fortunate 10%. The next step is probably to look at the abstract or introduction and find out whether our initial interest was justified. At this point we will probably only continue reading further when the paper seems closely related to our own work, either in terms of the method, result or context; this might be something like 50% of cases. So now we've discarded 95% of our original list; and the cull hasn't finished yet.

We'll probably pay this remaining 5% of authors the courtesy of reading their abstract, introduction and conclusion whilst flicking through at least some of the rest of the paper to see if we can get an overview of the details of their argument. Only if we can actually use the results or methods ourselves are we likely to get out hands dirty with the detailed science, which the author has spent months working through. This might happen in only a few cases – probably less than 1% of our original list.

So what does this insight tell us about writing our papers? Firstly, we don't want to miss out on the first cut because of a poor title. Secondly, even if we lure readers in with an informative abstract, very few people will read the entire article from start to finish. Clearly some people will avoid reading our work because it's too far removed from their own interests; but we should do our best to make it attractive to the others.

## 5.2 The title

Some hints and tips for a good title:

- Key words: identify key words that will interest your reader (perhaps from your introduction or abstract) and place them prominently in the title.
- Make a clear statement about your work, if it's possible to do so. Avoid deliberate ambiguity – it's annoying for the reader and doesn't look good when people cite your work.

- Omit symbols and notation where possible. Amongst other benefits, this avoids problems with keyword/bibdata searches and typesetting (emailed journal indexes and conference programmes don't usually support LaTeX).

## 5.3 Abstract, introduction and conclusion

More people will read these sections of your paper than any other. Furthermore, most of your audience will *only* read these sections. If something is not mentioned here, you have essentially hidden it.

You should therefore devote a significant proportion of your time to these sections of your work.

Peers (postgraduates in your research group) are a good choice to proof read these sections. While your supervisor's experience and close relationship with the material is useful for reading the detailed science, it is often a good idea to get a more objective opinion on these sections.

## 5.4 Overall structure

Make sure it's easy to locate key arguments in your work. Think about results you have cited – you wouldn't necessarily read the entire paper. Your introduction should provide a map of the key aspects of your work and, just as importantly, let the reader know where to find them.

## 5.5 Keeping the reader informed

As we have already seen, a few narrative comments, and informative use of emphasis, makes a technical writing much easier to follow.

## 5.6 A note on citations & plagiarism

This is something that research students tend to deal with better than undergraduates. Reading journal articles in your field should give you a decent understanding of which ideas need crediting (and to whom). The best piece of advice here is to be honest, recognise what you've contributed to a piece of work, and cite other people's work if and when you use it.

# 6 A Few Extra Points

This section is a somewhat random list of topics provided to address some questions raised by postgraduate students in the first session. Such questions are very welcome – particularly in sessions 1 & 2 so that I can prepare some examples and references.

In class we also turn to the summary of writing tips from Knuth's course, given as a handout in class.

**References**
Older style guides dedicate a lot of discussion to bibliographies. LaTeX makes them much easier and most authors should have little trouble following journal guidelines (See section 7 for more information).

**Hyphenated words**
Generally you will have an opinion as to whether compounded technical words are well enough established in your field to appear with or without a hyphen. A journal editor may disagree, but this is an issue that you can probably leave to them.

**We & I**
The word 'we' can be used to mean either 'the author and the reader' or 'the authors'. Substituting it as a formal alternative to 'I' is generally regarded as poor form. If the identity or the author is essential you may (sparingly) use 'I', otherwise write 'the author'. The reflexive form 'myself' should not be used to mean 'me'. Again, journal editors can advise on this matter.

**'etc.' and 'et al.'**
While both these Latin abbreviations can be read as 'and others' in English, the first refers to objects and the second to people.

**Examples**
We consider countable subsets of the real numbers, such as $\mathbb{Z}$, $\mathbb{Q}$, etc.

This result was proved by Smith et al.

**'i.e.' and 'e.g.'**
Don't confuse these abbreviations. 'i.e.' means 'that is' and 'e.g.' means 'for example'.

**Examples**
This is the set of real numbers strictly between 0 and 1 (i.e. the open unit interval).

We consider an open subset of the real numbers (e.g. the open unit interval).

**Paragraphs**
Mathematical arguments impose restrictions on paragraph structure but the following suggestions can often still be applied.

- Start paragraphs and sections with a good sentence (perhaps emphasising a key point in your argument);
- Each paragraph should correspond to one particular idea
- Where possible try to use the 'rule of 3', that is: state the idea, discuss the idea, and then re-state it in summary. An example of this is the 'state-prove-result' method we discussed for proofs.

**Writing numbers**
When using small numbers, but not talking about them, write them out in full; when referring to specific numbers, use numerals.

**Examples**
In this proof we consider two distinct cases.

There is the only one even prime: the number 2.

**Precise wording**
Be very careful when using vague words and phrases such as, 'well known', 'recent', 'usually'; particularly in mathematical arguments.

# 7 Writing Resources

Some of these books are useful as reference tools; others, like the web-book for this course, have sections that you can dip in and out of.. It's a good idea to make a note of any mistakes you're prone to making in your writing.

## 7.1 English dictionary

English dictionaries are readily available online. One good example is `http://dictionary.reference.com`, which provides search entries from a number of dictionaries.

## 7.2 Thesaurus

Very good if you find yourself over-using particular words. Again, these are readily available online, for example `http://thesaurus.reference.com`.

## 7.3 Bilingual dictionary

This is something probably best used in paper form. Online translators are notoriously problematic, and online reference dictionaries rarely provide as much disambiguation or usage guidance as the paper form.

## 7.4 English usage dictionaries

English usage dictionaries provide further clarification of tricky language issues. Examples of these are:

**M. Swan (1995), *'Practical English Usage, Second Edition'*, OUP;**

**H.W. Fowler (1965), *'A Dictionary of Modern English Usage, Second Edition'*, OUP.**

Beware: many online usage dictionaries and style guides are aimed at journalists, and are not of great use to scientists.

## 7.5 Style, punctuation, and grammar guides

**W. Strunk Jr., E.B. White and R. Angell (1999), *'The Elements of Style'*, Longman,** is an excellent short pamphlet covering the key principles of English writing. It has been updated as recently as 2007, although the 1918 version is still perfectly readable; the old version is available online, in several locations such as `sut1.sut.ac.th/strunk`.

**G. Jarvie (2007), *'Bloomsbury Grammar Guide, Second Revised Edition'*, A&C Black.** A word of warning: grammar guides can be tricky tools to use – often when you encounter a problem with your sentence construction you don't know the grammarian's name for it.

**L. Truss (2006),*'Eats Shoots and Leaves'*, Gotham,** is a recent guide to punctuation and its history. While it is an interesting read, it is not designed as a reference text.

***'The Chicago Manual of Style, 15th Edition'*, University of Chicago Press (2003),** is the standard style guide for editorial practice, although style does vary between journals (and on either side of the Atlantic).

## 7.6 Mathematical writing resources

**L. Gillman (1987),*'Writing Mathematics Well'*, MAA,** contains a similar range of topics to this course. It is an excellent manuscript, although currently out of print.

**D.E. Knuth, T.L. Larrabee and P.M. Roberts (1989), *'Mathematical Writing'*, MAA,** is a report of a course led by Don Knuth at Stanford. The text (excluding illustrations) is available online at `http://tex.loria.fr/typographie/mathwriting.pdf`.

### 7.7 Assignment specific guides

Other courses run by ESD address individual assignments: research articles, oral presentations, posters, the thesis, etc. These are extremely useful when you are preparing to write, or to re-draft your work. There is also a wide range of books on this subject.

### 7.8 Publishing guide

LaTeX guides, journal style guides, Powerpoint/Beamer guides, etc. are all readily available on the web (although the quality varies enormously).

## 8 Expository Papers

In order to see how some of the topics in this course can be put into practice, we look at a number of papers picked from the American Mathematical Monthly and the International Statistical Review. These are expository publications; so the material should be accessible to a diverse group of research students, and also provide suitable models for research articles.

We compare the titles, introductions and conclusions (or lack of) and pick out some examples of good and bad practice for some of the writing techniques we have already spoken about. We attempt to produce some much more effective titles for several of the articles, by removing symbols or making a precise statement about the article. We also discussed an applied mathematics paper, where a simple re-labeling of objects at the start could have avoided double-indexed quantities later on (and the occurrence of several degenerate matrices). Most of the pure mathematics articles would benefit from a few more introductory remarks and some extra explanation of the key steps in the argument. While the statistics papers tackled these issues better, and included an explicit list of keywords, the authors have struggled rather more with combining written text and symbols.

## 9 Assignment

In preparation for the second day of the course the postgraduate students are asked to prepare a short written assignment.

**Exercise 11**

Prepare a short piece of scientific writing for next week. We will be reviewing this work in groups, and the feedback from this will be more useful if the content is accessible to the majority of the class here today. The piece should be a couple of paragraphs, and at least no longer than a page.

- **Research summary or proposal:** This might be useful for progress reports.
- **Abstract or introduction:** Postgraduate students could use an old project/paper, or one in preparation. Otherwise they might like to try writing an alternative abstract to a published article, such as one from American Mathematical Monthly `www.maa.org/pubs/monthly.html`.
- **Short expository piece:** This might provide some interesting discussion in the second session. Following the style of the MicroEssay, write a short piece on a mathematical topic of your choice.

In the following week's class, each of the scripts is copied and distributed to small groups of postgraduates. We then begin the writing workshop, using the following exercise guide the analysis of each script. Postgraduate students are encouraged to leas the session themselves and use the course material, reference books and staff when needed.

**Exercise 12**

Make a note of the author and title of each piece. Read it through, making notes where appropriate, and try to provide answers to the following questions:

- What is the target audience for this writing?
- What are the keywords in the piece?
- Can you suggest an alternate title?
- Can you give a brief (one or two line) summary of the main argument?

The authors of each piece should then consider how well these answers match their own perceptions of the writing. They are then encouraged to lead any further discussion on the assignment.

## 10 Exercise Sheets

### Notes

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**Exercise 1**

Rewrite the following set without using symbols:

$\{f : \mathbb{R} \to \mathbb{R} \,:\, \forall x \in \mathbb{Q},\ f(x) \neq 0 \Leftarrow x \neq 0\}$.

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**Exercise 2**

Write a *coarse* and *fine* description of this object

$\frac{dy^2}{dx^2} - 3\frac{dy}{dx} - 2 = 0$

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**Exercise 3**
In groups, agree on a short description of the function (one graph handed out to each group). One of your group will then read out your solution and a member of another group will attempt to re-draw the graph from your definition.

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**Exercise 4**

Consider the expression,

$$\sin(\theta_1 - \theta_2) = \sin(\theta_1)\cos(\theta_2) - \cos(\theta_1)\sin(\theta_2).$$

Here are three possible ways that we could use the description to write about the object:

- This is a trigonometric identity. It is the formula for the sine of the difference of two angles.
- This is a trigonometric identity, the formula for the sine of the difference of two angles.
- This is a trigonometric identity, which gives us the formula for the sine of the difference of two angles.

Discuss the flow, emphasis and impact of each.

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**Exercise 5**
Study the following proof, then suggest how it might be improved for the reader.

**Proof:**

$$2 \nmid n \Rightarrow \exists k \in \mathbb{Z}\; n = 2k + 1.$$
$$n^2 - 1 = 4k(k + 1).$$
$$\forall k \in \mathbb{Z}\; 2 \mid k(k + 1). \;\square$$

We have therefore proved the following:
**Theorem:** $\forall n \in \mathbb{Z}, 2 \nmid n \Rightarrow 8 \mid n^2 - 1$.

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**Exercise 6**
Assess the merits of the following three descriptions of the set,

$\{x \in \mathbb{Q} : x \notin \mathbb{Z}, 1/x \in \mathbb{Z}\}$. [3 marks]

1. The set of reciprocals of rational numbers excluding zero and one.
2. The set of rationals such that the number itself is not an integer but its reciprocal is.
3. The set of reciprocals of integers so long as the integer is not zero and the integer does not have modulus one.

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**Exercise 7**
Provide a mark out of ten and some brief feedback for this essay (pass mark of 4/10 and first class boundary 7/10.)

**MicroEssay A**

The complex numbers form a field, under the usual addition and multiplication operations. Since they correspond to ordered pairs of real numbers, they can be represented by points in the plane. This is called the Argand diagram. Unlike the real numbers, the set of complex numbers is algebraically closed. Complex analysis, the study of complex functions, has produced a number of interesting results such as the Cauchy-Riemann equations and fractal sets such as the Mandelbrot set which you may have seen pictures of.

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**Exercise 8**
Provide a mark out of ten and some brief feedback for this essay (pass mark of 4/10 and first class boundary 7/10.)

**MicroEssay B**

In mathematics, a complex number is a number of the form

$$a + bi$$

where $a$ and $b$ are real numbers, and $i$ is the imaginary unit, with the property $i^2 = -1$. The real number $a$ is called the *real part* of the complex number, and the real number $b$ is the *imaginary part*. Complex numbers can be added, subtracted, multiplied, and divided like real numbers and have other elegant properties. In some fields (in particular, electrical engineering, where $i$ is a symbol for current), the imaginary unit $i$ is instead written as $j$, so complex numbers are sometimes written as $a + jb$.

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**Exercise 9**
Provide a mark out of ten and some brief feedback for this essay (pass mark of 4/10 and first class boundary 7/10.)

**MicroEssay C**

When solving a quadratic equation, it may happen that the discriminant be negative. If we still require that the equation has solutions, we are led to consider a quantity whose square is equal to $-1$, called the *imaginary unit.* Now, take two real numbers, multiply one of them by the imaginary unit, and add them together: it can be shown that any solution of a quadratic equation can be written in this way. The set of these expressions is called the set of complex numbers. It's an infinite set, equipped with the four arithmetical operations.

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**Exercise 10**
Provide a mark out of ten and some brief feedback for this essay (pass mark of 4/10 and first class boundary 7/10.)

**MicroEssay D**

Complex numbers are made out of two real numbers and the imaginary unit. The first real number is called the real part of the complex number and the second is the imaginary part. Add the real part to the imaginary part times the imaginary unit to give you the complex number. Mathematicians usually call the imaginary unit $i$ (engineers often call it $j$) so that a complex number looks like $z = a + ib$. The number can also be written in polar form using as the modulus times the exponent of the imaginary number times the angle.

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## 11 Graph handouts