cwork4b.tex 28/10/2013

MTH5117 Mathematical writing: Coursework 4

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DEADLINE: Sunday of week 6, at 23.55.

ASSESSED PROBLEMS [with allocated marks].
Problem 1: 2,3,4,5 [20]. Problem 2: 5, 9 [30].
Problem 3: 2, 3 [30]. Problem 4: 2 [20].

Problem 1. Describe each item accurately, with a short phrase. [$\not\in$]

1)	$f^{-1}(0)$	**2**)	$f(0)^{-1}$
3)	$f^{-1}(\{0\})$	**4**)	$f\vert_{\mathbb{Z}}$
5)	$(f \circ g)^{-1}$	6)	$f(\mathbb{R}) \cap \mathbb{Q}$.

Problem 2. Consider the symbolic sentences:

1. $\exists n \in \mathbb{Z},\ -n \notin \mathbb{Z}$
2. $\forall n \in \mathbb{N},\ 1/n \notin \mathbb{N}$
3. $\forall x, y \in \mathbb{R},\ xy = yx$
4. $\forall n \in \mathbb{N},\ \sqrt{n} \in \mathbb{R} \setminus \mathbb{Q}$
5. $\forall n \in \mathbb{Z},\ 2 \,|\, n(n+1)$
6. $\exists x \in \mathbb{R},\ e^x \in \mathbb{Q}$
7. $\forall n, m \in \mathbb{Z},\ (2 \not| n \wedge 2 \not| m) \Rightarrow 2 \,|\, (m+n)$
8. $\forall x, y \in \mathbb{R},\ (x < y) \Rightarrow (x^2 < y^2)$
9. $\forall y \in \mathbb{Z},\ \exists x \in \mathbb{R},\ \log(x) = y.$

For each sentence

(a) State whether it's true or false.
(b) State it with words. [$\not\in$]
(c) State its negation with symbols.
(d) State its negation with words. [$\not\in$]

Be concise.

Problem 3. Each function definition contains an error. Explain what is the error, and how it should be corrected.

1. $f : \mathbb{R} \to \mathbb{R} \qquad x \mapsto \dfrac{1}{x^2 + x - 1}$
2. $f : \mathbb{R} \to \mathbb{R} \qquad x \mapsto \sqrt{x^2 - 1}$
3. $f : \mathbb{N} \to \mathbb{N} \qquad z \mapsto \#\left\{\dfrac{a}{b} \in (0,1) \,:\, b \leqslant n\right\}$
4. $f : \mathbb{Z} \to \mathbb{Z} \qquad n \mapsto n\mathbb{Z} \cap (n^2 + 1)\mathbb{Z}.$

Problem 4. Answer the questions as clearly as you can.
[*You may use symbols. Consider establishing some notation.*]

1. Let A and B be sets. Why are the sets $(A \setminus B)^2$ and $A^2 \setminus B^2$ not necessarily equal? Under what conditions are they equal?
2. Let $f : X \to Y$ be a function, and let A be a subset of X. Why are the sets A and $f^{-1}(f(A))$ not necessarily equal? Under what conditions are they equal?