MTH5117 Mathematical writing: Coursework 3

Franco Vivaldi

DEADLINE: Sunday of week 5, at 23.55.

ASSESSED PROBLEMS [with allocated marks].
Problem 1:   6, 7 [20].   Problem 2:   3, 5 [20].
Problem 3:   5, 6, 7, 8 [40]. Problem 4:   1, 2 [20].

Problem 1.   Write each existence statement with symbols, using the quantifier $\exists$ in each case.

1. The integer n is odd.

2. The set X has more than one element.

3. The sets X and Y have non-empty intersection.

4. The unit circle has a rational point.[1]

5. The equation $f(x) = 0$ has a positive integer solution.

6. $z \in f(X)$.

7. The integers m and n are not coprime.

8. The integer n is not divisible by 3.

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[1]See web-book.

Problem 2. Write each existence statement with symbols, using the quantifier $\forall$ (and, if appropriate, $\exists$), in each case.

1. The equation $f(x) = 0$ has no real solution.

2. The sets A and B are disjoint.

3. The open unit interval has no greatest element.

4. The set X has a greatest element.

5. The function $f : X \to Y$ is constant.

6. Eventually, all terms of the integer sequence $(a_1, a_2, \ldots)$ become negative.

Problem 3. Write each symbolic sentence in two ways:

$i)$ without any symbol, apart from f.

$ii)$ with symbols only, using quantifiers. (You may assume that $f : \mathbb{R} \to \mathbb{R}$.)

1.	$f(\mathbb{R}) = \mathbb{R}$	2.	$\# f(\mathbb{R}) = 1$
3.	$0 \in f(\mathbb{Z})$	4.	$f(\mathbb{R}) \subset \mathbb{Q}$
5.	$f(\mathbb{Z}) = \{0\}$	6.	$f^{-1}(\mathbb{Z}) = \emptyset$
7.	$f(\mathbb{R}) \supset \mathbb{Z}$	8.	$f(\mathbb{Q}) \cap \mathbb{Q} = \emptyset.$

Problem 4. Explain, clearly and concisely. [$\notin$, 30]

1. What is the difference between an equation and an identity?

2. What is the difference between an ordered pair and a set with two elements?