

MTH5117 Mathematical writing: Coursework 2

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DEADLINE: Sunday of week 4, at 23.55.

*ASSESSED PROBLEMS [with allocated marks].**Problem 1: 5, 7 [20]. Problem 2: [20].**Problem 3: 4, 5 [20]. Problem 4: 4, 5 [20].**Problem 5: 3 [20].*

Problem 1. For each item, provide two levels of description. (Same prescription as problem 3, cwork1.)

1. $(a_k), \quad a_k \in \mathbb{Q}, \quad a_k \geq 0$
2. $\sum_{k=1}^N z_k, \quad z_k \in \mathbb{C}, \quad |z_k| = 1$
3. $\frac{d}{dx} [f(x) + g(x)] = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$
4. $e^{ix} = \cos(x) + i \sin(x)$
5. $\int_{-\infty}^{\infty} F(x, y) dx$
6. $f(x) \stackrel{\text{def}}{=} \sum_{k=0}^{\infty} a_k x^k, \quad a_k = \frac{1}{1 + k^2}$
7. $\sum_{k=1}^3 \frac{\partial F(x_1, x_2, x_3)}{\partial x_k}$.

Problem 2. Explain, clearly and plainly.

I have three complex numbers, and I must decide if they lie on the same line in the complex plane. What shall I do? [\neq , 50]

[*The numbers are not necessarily distinct.*]

Problem 3. The following expressions define sets of real functions $f : \mathbb{R} \rightarrow \mathbb{R}$. Turn words into symbols, denoting by \mathcal{R} the set of real functions.

[*Use the Zermelo definition, and —where appropriate— the quantifier \forall . The meaning of ‘vanish’ is explained in the web-book.*]

1. The set of positive real functions.
2. The set of real functions that do not vanish at the origin.
3. The set of real functions that vanish identically in the open unit interval.
4. The set of real functions that are positive for negative values of the argument.
5. The set of real functions that vanish at all even integers.
6. The set of real functions that may become zero only at the rationals.

Problem 4. Let \mathcal{R} denote the set of real functions. Turn symbols into words. [✓]

1. $\{f \in \mathcal{R} : f(0) = 0\}$
2. $\{f \in \mathcal{R} : \forall x \in \mathbb{Q}, f(x) = 0\}$
3. $\{f \in \mathcal{R} : \forall x \in \mathbb{R}, x < 0 \Rightarrow f(x) = 0\}$
4. $\{f \in \mathcal{R} : \forall x \in \mathbb{R}, f(x) \in \mathbb{Z}\}$
5. $\{f \in \mathcal{R} : \forall x \in \mathbb{Q} \setminus \{0\}, f(x) \neq 0\}$
6. $\{f \in \mathcal{R} : \forall x \in \mathbb{R} \setminus \{0\}, xf(x) > 0\}$.

Problem 5. The following boolean expressions are false. State them with words, and then explain why they are false. [✓, 40]

1. $\forall z \in \mathbb{C}, \bar{z} \neq z$
2. $\forall x \in \mathbb{R} \setminus \{0\}, \frac{d|x|}{dx} = 1$
3. $\forall x, y \in \mathbb{R}, |x + y| < |x| + |y|,$
4. $\forall a \in \mathbb{R}, \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}.$

[In part 1, what is \bar{z} ? In part 4, find an appropriate word for the symbol a .]