Queen Mary & Westfield College
UNIVERSITY OF LONDON

MAS/103 COMPUTATIONAL MATHEMATICS I

Thu May 27 1999, 10:00

Duration: 3 hrs.

You should attempt all questions. Marks awarded are shown next to the questions. Write your answers in the form of Maple commands, unless instructed otherwise. CALCULATORS ARE NOT PERMITTED.

[**1**] *Basic literacy (10 marks).*

Define the rational numbers, and describe their significance and main properties. Use approximately 100 words and no mathematical or Maple symbol whatsoever.

[**2**] *Basic numeracy (12 marks).*

(a) Determine BY HAND the fractional part of 1182/87, in reduced form.

(b) Simplify BY HAND the following expression

$$-\frac{2}{3\,x^2}\left[\left(x^2-\frac{1}{3}y\right)^2-\frac{1}{9}y^2\right]^2-\frac{1}{9}x^2y\left(8x^2-\frac{8}{3}y\right).$$

(c) Simplify BY HAND the following expression

$$\frac{1}{\sqrt{5}}\,\frac{\sqrt{30}-\sqrt{12}\,\sqrt{15}}{(\sqrt{2}-\sqrt{3})^2}$$

to the form $m+n\sqrt{d}$, where m,n and d are integers.

[**3**] *Sequences (12 marks).*

Consider the sequence

$$a_n = \frac{n - 2^{-n}}{2 + \dfrac{1}{3^{2n}}} \qquad\qquad n \geq 0.$$

(a) Construct a user-defined function `a(n)` whose value is a_n.

(b) Plot the elements $a_0, \ldots, a_{30}$, as points on the (n, a)-plane.

(c) Show that the integer part of a_{30} is equal to 14. (Indicate explicitly what output you expect from Maple.)

[**4**] *Digits (12 marks).*

(a) Display 50 decimal digits of e^π.

(b) Show that 100! has 158 decimal digits.

(c) Show that the third decimal digit of 17/97 is odd.

[**5**] *Sums and products (15 marks).*

(a) Let p_k be the kth prime number, and let $\binom{n}{k}$ be the binomial coefficient. Compute

$$\sum_{k=0}^{10} \binom{p_{k+1}}{k}$$

displaying the result in factored form.

(b) Consider the sequence of rational functions

$$f_n(x) = \frac{1}{x-1}\prod_{k=1}^{n}\left(1-\frac{k}{x^k}\right) \qquad\qquad n = 1, 2, \dots.$$

i) Construct a function `f(n)` for $f_n(x)$.

ii) Using the above function, construct $f_4(x)$, expressing it in the form

$$\frac{(x^2-2)^2\,(x^3-3)\,(x^2+2)}{x^{10}}$$

iii) Transform the numerator of the above expression into

$$(x^2-2)^2\,x^5 - 3\,(x^2-2)^2\,x^2 + 2\,(x^2-2)^2\,x^3 - 6\,(x^2-2)^2.$$

[**6**] *User-defined functions (19 marks).*

(a) Construct the boolean characteristic function `pr(x)` of the rational numbers x whose numerator and denominator are both prime.

(b) Construct the characteristic function of the integers that are relatively prime to 30, whence, by performing a suitable summation, determine how many such integers lie between 1000 and 1100.

(c) Let `S` be a finite (possibly empty) set of integers. Consider the following statements

```
> h:=x->evalb(x>=0):
> map(h,S):
> " intersect {true}:
> nops(");
```

The last expression can assume only two values. Explain what they are, and what information they provide about the set `S`.

(d) Let L be a list. Contruct a function `rep(L)` whose value is *true* if L has repeated elements, and $false$ otherwise.

[**7**] *Iteration (20 marks).*

(a) Using a `do`-loop, construct the nested expression

```
[[[[[[1],2],3],4],5],6]
```

No intermediate result should be displayed.

(b) Determine the smallest positive integer n for which the nth prime is greater than $10\,n$.

(c) Consider the recursive sequence of matrices

$$B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}; \qquad A_0 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}; \qquad A_{t+1} = A_t B - A_0 \qquad t \geq 0,$$

Compute

$$\sum_{t=1}^{20} \det(A_t).$$

(d) Two primes are *consecutive* is there is no prime between them, i.e., 7 and 11. Compute the number of primes which are equal to 2 plus the product of two consecutive primes, each smaller than 10^4.
[*Hint:* such primes are of the form $2 + p_n p_{n+1}$, where p_n is the nth prime, and n lies in a suitable range.]

End of examination paper