MAS/103 Computational Mathematics I: Coursework 7

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This coursework will be assessed and count towards your final mark for the course

DEADLINE: Wednesday of week 11, at 1:00 pm.

CONTENT: Iteration.

PREREQUISITES: Section 8.1 of lecture notes.

M⋆icroESSAY: Explain what is a `do`-loop, in fewer than 50 words and without using any Maple symbol (apart from `do`) .

Problem 1. In the following exercises you must use the `do`-structure.

(a) Display the reciprocal of the first 5 positive integers. Hence do the same using the function `seq`.

(b) Compute the sum of the first 10 prime numbers. (When you are sure that your code is working, suppress all intermediate output within the loop.) Hence check your result with the function `add`.

(c) Compute the product of the first 10 prime numbers. Hence check your result with the function `mul`.

(d) Let

$$p(x) = x\,(3 + x\,(6 + x\,(9 + x\,(12 + x\,(15 + x))))).$$

Show that $p(x)/x$ is irreducible.

Problem 2. Consider the following recursive sequence of rationals

$$x_0 = 0; \qquad x_{t+1} = f(x_t) = \frac{x_t - 2}{x_t + 2}, \qquad t \geq 0 \tag{1}$$

(a) Compute the element x_{10}, without displaying any intermediate output.

(b) Compute the first 10 elements of the sequence (1) (that is, x_1 to x_{10}), displaying the value of t and x_t at each step.

(c) Compute the sum of the first 10 elements of the sequence (1).

(d) Plot the elements of the sequence (1) for t in the range $30, \ldots, 100$, connecting points with segments.

Problem 3. In the following exercises you must use the `while` option of the `do`-structure:

(a) Display the positive cubes smaller than 500.

(b) Compute the smallest integer greater than 1 which is relatively prime to 9699690.

(c) Compute the smallest integer $n > 1$ for which $n^6 - 1$ is divisible by 79.

(d) Compute the smallest positive integer n for which $n^2 + n + 41$ is *not* prime.

(e) Compute the smallest positive n which is a multiple of 3, and for which $2^n > n^4$.

(f) Consider the following recursive sequence of complex numbers

$$z_0 = -2\,i; \qquad\qquad z_{t+1} = z_t^2 + \frac{4}{3}\,i.$$

Compute the smallest value of t for which $|z_t| > 10^5$.

Problem 4. We consider the number q_n of composite integers following the nth prime p_n. Thus $q_4 = 3$, because the 4th prime $p_4 = 7$ is followed by the 3 composite integers $8, 9, 10$. For $n > 1$, q_n is always odd (make sure you believe this).

(a) Compute the smallest prime p_n which is followed by at least 9 composite integers. Display also the corresponding value of n.

(b) Let a_t be the smallest value of n for which $q_n \geq t$. Thus p_{a_t} is the smallest prime which is followed by at least t composite integers. (Think about it. Read the definition again. In the previous problem you have computed a_9 and p_{a_9}.) Compute and display t, a_t and p_{a_t}, for all odd t smaller than 20 (excluding $t = 1$).

◇ *MAPLE CHALLENGE: (for top marks)*

Problem 5. We define an infinite recursive sequence $S = s_1, s_2, \ldots$ of elements of the set $\{1, -2\}$. We first let $S = 1, -2, 1$, and then we apply repeatedly the substitution: $S \mapsto S, 1, S, -2, S, 1, S$. After t substitutions, S will have $4^{t+1} - 1$ elements. We then define the sequence

$$\sigma_0 = 0 \qquad \sigma_n = \sum_{k=1}^{n} s_k \qquad n \geq 1.$$

Plot the first 4^4 elements of the sequence $0, z_0, z_1, z_2, \ldots$, where z_n are complex numbers defined as

$$z_n = \sum_{k=0}^{n} e^{\pi\, i\, \sigma_k / 3}.$$

Connect points with segments. Use the plot options `axes='none',scaling='constrained'` to improve visualization. The result is the image of *fractal* called *Koch's snowflake*.