MAS/103 Computational Mathematics I: Coursework 5

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This coursework will be assessed and count towards your final mark for the course

DEADLINE: Wednesday of week 8, at 1:00 pm.

CONTENT: Structure of expressions. Polynomials and rational functions.

PREREQUISITES: Chapters 5 and 6 of lecture notes.

M⋆croESSAY : Write a synopsis of the Maple function `map`, in fewer than 50 words. You may not use any Maple symbol, apart from `map`.

Problem 1. Consider the polynomial $(x^2-1)(x^4+x^2+1)$. Using Maple, transform it into:

a) $(x^2-1)x^4+(x^2-1)x^2+x^2-1$

b) $(x^4+x^2+1)x^2-x^4-x^2-1$

c) $(x-1)(x+1)(x^2+x+1)(x^2-x+1)$

d) $(x-1)x^5+(x-1)x^4+(x-1)x^3+$
$+(x-1)x^2+(x-1)x+x-1$

e) x^6-1

f) $(x-1)(x^5+x^4+x^3+x^2+x+1)$

Problem 2. Consider the rational expression

$$x+\cfrac{1}{x^2+\cfrac{1}{x^3+\cfrac{1}{x^4}}}$$

(a) Transform it into

$$\frac{x^{10}+x^3+x^5+x^7+1}{x^2(x^2+x+1)\,(x^5-x^4+x^2-x+1)}$$

(b) Transform the latter into

$$\frac{x^{10}+x^3+x^5+x^7+1}{x^9+x^2+x^4}$$

(c) Transform the latter into

$$x\,+\,\frac{x^7+1}{x^9+x^2+x^4}$$

Problem 3.

(a) Construct the boolean characteristic function `chi(p)` of the polynomials $p(x) \in \mathbf{Z}[x]$ which are divisible by $x + 2$.

(b) Consider the sequence of polynomials in $\mathbf{Z}[x]$

$$p_n(x) = x^2 + (n - 1)\,x + n, \qquad n \geq 1.$$

Use the funtion `chi(p)` to determine the the smallest value of n for which $p_n(x)$ is divisible by $x + 2$.

Problem 4. Each expression `E` on the left is transformed into the expression on the right by `map(f,E)`, for a suitable function `f`. Construct such user-defined function `f` in each case.

	E	map(f, E)
a)	$[0, 1, 2, 3]$	$[100, 101, 102, 103]$
b)	$a + b + c + d$	$1/a + 1/b + 1/c + 1/d$
c)	$\{-20, 10, -10, -30, 20\}$	$\{10, 20, 30\}$
d)	$[0, 1, 1, 1, 0, 1, 0, 1]$	$[1, 0, 0, 0, 1, 0, 1, 0]$
e)	$[a + b, c + d, e + f]$	$[a, b, c, d, e, f]$
f)	$[a + b, c + d, e + f]$	$[ab, cd, ef]$
g)	$[d^2, a^3, b^4, a^7]$	$[d/(2 + d),\ a/(3 + a),\ b/(4 + b),\ a/(7 + a)]$
h)	$[[a], [[b]], [[[c]]], [[[[d]]]]]$	$[a,\ b,\ [c],\ [[d]]]$
i)	$[[a], [d, e, f], [g, h, i, j], [b, c]]$	$[1, 3, 4, 2]$
j)	$[[a], [b, c], [d, e, f, g], [h, i, j]]$	$[a, c, g, j]$

Problem 5. Let `L=[a,b,c,...]` be a list of data to be plotted. Construct a function `PlotList(L)` which returns the list `[[1,a],[2,b],[3,c],...]` required by `plot` to display `L`.

◇ *MAPLE CHALLENGE: (for top marks)*

Problem 6. Let $f(x)$ be a polynomial. A number α is a *root* of $f(x)$ if $f(\alpha) = 0$. Thus $\sqrt{2}$ is a root of $f(x) = x^2 - 2$, and so is $-\sqrt{2}$. For any positive integer n we let

$$f_n(x) = x^n - 1; \qquad \alpha_{n,k} = e^{2\pi i k/n} \qquad k = 0, 1, \ldots, n - 1.$$

Then $\alpha_{n,k}$ is a root of $f_n(x)$ for any choice of k, because

$$f_n(\alpha_{n,k}) = \alpha_{n,k}^n - 1 = e^{2\pi i k} - 1 = 1 - 1 = 0.$$

Now let $n = 15$. The polynomial $f_{15}(x)$ factors into the product of irreducible polynomials, the largest of which has degree 8. Determine the 8 values of k corresponding to the roots of the latter. [To get full marks you must not perform floating-point calculations.]