MAS/103 Computational Mathematics I: Coursework 4

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This coursework will be assessed and count towards your final mark for the course

DEADLINE: Wednesday of week 7, at 1:00 pm.

CONTENT: Digits of rationals. Real and complex numbers.

PREREQUISITES: Chapter 4 of lecture notes.

MicroESSAY: Explain what are the complex numbers. Use fewer than 50 words and no mathematical or Maple symbol whatsoever (not even i).

Problem 1.

(a) For each of the following rational numbers

$$i)\ \frac{10001}{11110} \qquad ii)\ \frac{9}{17} \qquad iii)\ \frac{1}{151552}.$$

determine the period m of the pre-periodic decimals, and the period n of the repeating decimals.

(b) For the case *ii)* determine the third digit of r using only the functions `irem` and `iquo`.

Problem 2. The *mediant* of two rational numbers $x = a/b$ and $y = c/d$ in reduced form, is defined as $(a+c)/(b+d)$. The mediant of x and y lie in $[x, y]$ —the interval with endpoints x and y. (Can you prove it?)

(a) Construct a function `med(x,y)` whose value is the mediant of x and y.

(b) Let $\beta = \sqrt{2} - 1$. (You may assume that β is not rational.) Show that β lies in $[0, 1]$.

(c) Compute the mediant r of 0 and 1, using `med`, hence decide which of the two sub-intervals $[0, r]$ or $[r, 1]$ contains β. (Convince yourself that $\beta \neq r$, because r is rational and β is not.) Then compute the mediant of that interval, and decide which of the two resulting sub-intervals contains β, and so on. Repeat this process until you have determined a rational number whose distance from β is less than 10^{-2}.

Problem 3. Let $i = \sqrt{-1}$.

(a) Let $z = 17 - 23i$. Compute, with Maple

$$1/z, \quad \bar{z}, \quad |z|, \quad \arg(z), \quad \mathrm{Re}(z), \quad \mathrm{Im}(\bar{z}), \quad \overline{1 - \bar{z}^2}.$$

where $\bar{z}$ is the complex conjugate of z.

(b) Consider the following recursive sequence of complex numbers

$$z_0 = -\frac{4}{41}(9 + i); \qquad z_{t+1} = 8\,\frac{z_t - i}{i\, z_t + 10} \qquad t \geq 0.$$

Show that z_1, z_2 and z_3 lie on a circle centered at the point $z = 4 + i$, and determine its radius.

(c) Let z_t be as above. Determine the smallest value of $t \geq 3$ for which z_t lies inside a circle of diameter $1/10$ centered at $-2i$.

Problem 4. Let the complex numbers $z_1 = -91 - 24i$ and $z_2 = 200 + 100i$ be two vertices of a square in the complex plane. There are three such squares (think about it).

(a) Determine the area of each of the three squares.

(b) Construct the three squares and plot them, all in the same plot. (Maple may scale the two axes differently, resulting in a distorted picture. To prevent this from happening, insert the option `scaling=constrained` as the last argument of the function `plot`.)

◇ *MAPLE CHALLENGE: (for top marks)*

Problem 5. Construct a function `a(n)` whose value is the nth term of the following sequence ($n \geq 1$)

$$1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, \ldots$$