

MAS/103 Computational Mathematics I: Coursework 1

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This coursework will be assessed and count towards your final mark for the course

DEADLINE: Wednesday of week 3, at 1:00 pm.

CONTENT: Expressions. Integers and rationals.

PREREQUISITES: Sections 1.1–1.5 of Lecture Notes

MicroESSAY : Explain what is an arithmetical expression, an algebraic expression, and an assignment statement in Maple. You must use fewer than 50 words, and no mathematical or Maple symbol. The essay must be typeset in a Maple worksheet.

Problem 1. Construct the following expressions:

$$1) \frac{ab}{c} \quad [0]; \quad 2) \frac{a}{bc} \quad [1]; \quad 3) a^{b+c} \quad [1]; \quad 4) a^b - \frac{c}{d} \quad [0].$$

$$5) \frac{a^{bc}}{d} \quad [1]; \quad 6) \frac{a^b - c}{d} \quad [1]; \quad 7) \frac{a-b}{c-d} \quad [2]; \quad 8) \frac{a}{b + \frac{c}{d}} \quad [1].$$

Try to minimize the use of parentheses by taking advantage of the order of evaluation of arithmetical operators. The minimal number of parentheses needed in each case is indicated in square brackets.

Problem 2. Consider the following expression

$$a + \frac{1}{b + \frac{1}{a + \frac{1}{b + \frac{1}{a}}}} \quad (1)$$

- Construct it by means of a single Maple expression.
- By repeated use of the ditto variable "`:=`", construct it recursively, in four or five steps. [*Hint:* begin from the end.]
- Use the function `subs` to evaluate (1) for $a = b = 1$, and then for $a = 10$ and $b = 20$.

Problem 3. Construct the following expression

$$1 + z + z^2 + z^3 + z^4.$$

Then use the function `subs` to evaluate the sum

$$1 + 2 + 2^2 + 2^3 + 2^4 - 1 - 3 - 3^2 - 3^3 - 3^4 + 1 - 4^2 + 4^4 - 4^6 + 4^8.$$

Problem 4. Let $a = 5001999958$ and let $b = 5004400966$.

- (a) Compute the greatest common divisor d of a and b .
- (b) Verify the following equation and inequalities

$$a = bq + r \quad 0 \leq r < b$$

where q is the quotient and r is the remainder upon dividing a by b .

- (c) Verify that Maple represents a/b in reduced form. You must check that a/b is displayed as a'/b' where

$$a' = \frac{a}{d} \quad b' = \frac{b}{d}$$

and d is as above. The integers a' and b' are extracted from the value of a/b by means of the Maple functions `numer` and `denom`, respectively.

- (d) Find a proper divisor of a that does not divide b .

Problem 5. In the following exercises you should not display any digit. Display only the minimal output required to establish your result.

- (a) Place the following integers in ascending order

$$100^{100}, 80^{120}, 60^{140}, 40^{160}, 20^{180}.$$

- (b) Find the largest integer whose 17th power is smaller than 10^{30} . (Think about it.)

Problem 6. Let

$$x = \frac{89}{144} \quad y = \frac{377}{610} \quad z = \frac{987}{1597}.$$

- (a) Consider the triangular inequalities

$$(i) \quad |x - z| \leq |x - y| + |y - z| \quad (ii) \quad |x - y| \leq |x - z| + |z - y|.$$

Verify that strict inequality holds in case (i), and equality holds in case (ii).

- (b) Find a rational number r lying between x and z , which is closer to z than y is. The denominator of r should be as small as possible. Explain your strategy clearly and concisely.

◇ *MAPLE CHALLENGE: (for top marks)*

Problem 7. Find a rational r as in the previous problem, with no more than 4 decimal digits in the denominator. There is a solution with 3 digits: can you find it?

- Add your student number, surname and initials at the top of the worksheet. Save the worksheet under a suitable filename (e.g., `cwork1.mws`), then print it, and hand it in.