cwork5b.tex 4/11/2003

MAS/202 Algorithmic Mathematics: Coursework 5

Franco Vivaldi

DEADLINE: Wednesday of week 7, at 12:00 pm.

CONTENT: Polynomials

M*icroESSAY : Write an essay on polynomials [∉, 100]

Problem 1. We represent a polynomial over a commutative ring R as a finite sequence C of elements of R, which specifies the coefficients of the polynomial, starting from the constant term. The elements of C are $C_1, C_2, \ldots$, and its cardinality is $\#C$.

Write an algorithm to the following specifications

`Algorithm Degree`
`INPUT`: C, a finite sequence of elements of R.
`OUTPUT`: d, where d the degree of the polynomial represented by C.

[*Hint:* pay attention to the mismatch between subscripts, and to the fact that C may have zero entries.]

Problem 2. In the statements below, let m and n assume integer values, and let `W` denote the while-loop.

$x := m$;
$y := n$;
$z := 1$;
`while` $y \neq 0$ `do`
    $z := z \cdot x$;
    $y := y - 1$;
`od`;

(a) For which integer values of n will `W` terminate? Explain your answer.

(b) Prove that the boolean expression $x^y z = m^n$ is a loop invariant for `W`. [*Hint:* read the web-book.]

(c) Suppose that `W` terminates. Show that, on termination, $z = m^n$.

Problem 3. Let $R = \mathbb{Z}/(9)$, and let $a = 3\,x^3 + 3\,x + 2$, $b = 6x^3 + 1$ be polynomials in $R[x]$. Determine

$$\deg(a^2), \quad ab, \quad 3\,a, \quad a + b, \quad b^3.$$

Problem 4. Let $R = \mathbb{Z}/(7)$, and let $a = 5x^3 + 4x + 1$, $b = 2x^2 + x + 6$ be polynomials in $R[x]$. Use the algorithm `PolynomialQuoRem` to determine

$$a \,\mathrm{DIV}\, b, \quad a \,\mathrm{MOD}\, b, \quad b \,\mathrm{DIV}\, a, \quad b \,\mathrm{MOD}\, a.$$