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Mini-course: 'Expansions in non-integer bases'

Problem Sheet 2

1. Prove that $L(\sqrt{2}) = 0$. (*Hint: consider an arbitrary sum $\sum_{k=1}^{n} a_k (\sqrt{2})^k$ and separate the odd and even powers.*)

2. Prove that $L(\beta) = 1$ for any β between the golden ratio and 2. (*Hint: consider the subsequence $y_n = \beta^2 + \beta^4 + \cdots + \beta^{2n}$ and try to find its neighbours.*)