ERGODIC THEORY AND ARITHMETIC DYNAMICS

Franco Vivaldi
Queen Mary, University of London

The emergence of a dynamical systems perspective in number theory, and the parallel growth in the use of algebraic and arithmetical methods in dynamics, constitute a recent interdisciplinary development of great significance. The interaction of these disciplines identifies a multi-faceted area of research —loosely labelled *Arithmetic Dynamics*— which is rapidly expanding, has great potential for cross-fertilisation, and is rich in applications. The publication of over 250 research articles (half of which in the past 10 years), and of three research monographs[1] indicates that this subject area is approaching maturity[2].

We believe that time is ripe for this knowledge to percolate down to the early stages of postgraduate training. The present course will help the students achieve a synthesis of dynamics, ergodic theory, and arithmetic at an accessible level.

Ergodic-theoretic material seldom fits within undergraduate curricula. Yet, advanced applications of the theory of dynamical systems both within and outside mathematics (computer science, electronic engineering, physics), increasingly require understanding of ergodic theory, which provides the organising principles for the study of chaos and complex phenomena. At the same time, the methods of arithmetic, and, more generally, of discrete mathematics, play an increasing role in dynamical systems research, a development stimulated by the advent of computers.

A student of number theory is not expected to have developed a background in dynamical systems. Yet, dynamical and ergodic-theoretic ideas have been used in arithmetic for a long time (beginning with Gauss' work on the digits of rationals and continued fractions). More recently, these methods have seen widespread applications. Thus deep results on distributional properties of various arithmetical quantities have been proved using ergodic theory. Furthermore, dynamical systems over arithmetical sets (algebraic number fields, finite fields, p-adic fields, algebraic varieties, etc.), are objects of intense research.

This course brings these two aspects of postgraduate education under a single heading. Arithmetical phenomena are concrete and tangible; they provide compelling motivations, and give an opportunity to see meaningful ergodic theory in action. Combining the two perspectives will consolidate knowledge and give new insights.

---

[1] See `http://www.maths.qmul.ac.uk/~fv/database/algdyn.pdf`.

[2] The 2010 Mathematical Sciences Classification Scheme includes the new entry: *37Pxx Arithmetic and non-Archimedean dynamical systems.*