Queen Mary
University of London

**Goldsmiths' Company
Mathematics Course 2014**

School of Mathematical Sciences

Mathematical Computing Workshop

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Getting started

- This is a self-paced workshop but **please ask for help** whenever you need it.
- If necessary, start your computer and select the Windows environment.
- Log in using the **Maths** username / password information on your name badge.
- Start Maple by double-clicking on the Maple icon on the desktop.
- Wait while Maple starts and then select **Worksheet** mode to open a new empty worksheet.

Some key points to remember

- Maple is case-sensitive.
- The Expression and Common Symbols palettes should suffice to enter most expressions. The Expression palette provides templates containing coloured placeholders that you should replace with the correct values. You can use *Tab* and *Backtab* (*Shift-Tab*) to move between placeholders, or you can select them with the mouse. You can insert another template into a placeholder to build up complicated expressions.
- Type a space for implied multiplication, * for explicit multiplication (or use ✕ from the Common Symbols palette), / to enter a denominator and ^ to enter a power. Press the right-arrow key or click the mouse to the right of the current expression to exit a denominator or power.
- Use only parentheses (round brackets) for function arguments and expression grouping. Square brackets denote lists and curly braces denote sets. A function name must be followed **immediately** by an opening parenthesis; **do not include any space** (which turns the function application into an implied product).
- Right clicking on Maple output (or input) gives a list of context-dependent options that can be used to manipulate the expression. You can also re-use output via its "equation label" with *Insert > Label...* or *Control-L*.
- Press the *Enter* / *Return* key to evaluate or execute your input and move the cursor to a new input prompt.
- Try the Help menu if you get stuck; Maple contains a **huge** amount of information (including a mathematics dictionary) accessible via the help system. You may find Quick Reference and Quick Help particularly useful for navigating the user interface, and the general Maple Help (also available via the right-most button on the toolbar) useful for more mathematical help.
- It is safest to execute a *restart* command before you begin each new answer (provided it is unrelated to previous answers), i.e. type

> *restart*

and then press the *Enter* / *Return* key.

• If in doubt, experiment and/or ask for help!

Questions

Attempt whichever of the following questions you like. They are unrelated, so you can attempt them in any order. They are based loosely on Introduction to Mathematical Computing, a first-semester Mathematical Sciences module at Queen Mary. To answer a question, enter the appropriate input at the next input prompt in the worksheet you have just opened

> *like this*

and then press the *Enter / Return* key to see the result.

Example solutions

To see my example solutions, open this document from the course website in Maple by clicking on the *File* menu, selecting *Open URL…* and typing either the shortened URL **goo. gl/zGqSr1** (which will open via a web browser) or the full URL

www.maths.qmul.ac.
uk/~fjw/goldsmiths/2014/FrancisWright/Mathematical_Computing_Workshop.mw

By default, this will give you a new tab within the same Maple application. If you prefer, you can right click on the new tab and select **Open Tab in New Window**. You can then tile the two windows (whereas you cannot tile documents within the same window). Alternatively, browse to the URL and open it from a web browser.

Click on the triangle preceding a solution to expand it. **Execute each step of the solution to see how it works by clicking on the restart command and then pressing the *Enter / Return* key repeatedly.**

▼ Polynomial algebra: square-free factorization

Enter and evaluate the assignment $f := x^6 - 14\,x^5 + 80\,x^4 - 238\,x^3 + 387\,x^2 - 324\,x + 108$. Now compute the full square-free factorization of f by using only differentiation, GCD and division, as outlined in my presentation. You can do this as follows.

Compute the square-free factor, i.e. the product of all distinct irreducible factors, as $s1 := normal\left(\dfrac{f}{gcd(f, f')}\right)$. Divide this out of f to leave $f1 := normal\left(\dfrac{f}{s1}\right)$. Now repeat the process to compute the square-free factor of $f1$ as $s2 := normal\left(\dfrac{f1}{gcd(f1, f1')}\right)$. This is the product of all the distinct irreducible factors of f that appear to the power of at least 2, which must also be a factor of $s1$. So dividing $s1$ by $s2$ gives the square-free factors of f that appear to the power 1 but no higher. Continue this process systematically to give the full square-free factorization of f, which for this simple example is also the irreducible factorization. Check it by applying *expand* to the factorized expression for f and/or applying *factor* to f.

▶ *Solution*

Calculus

(a) Enter and evaluate the definite integral $\int_{-\infty}^{\infty} e^{-x^2}\, dx$, then approximate the result numerically. (Hint: use the Expression palette to enter the integral and the exponential, and the appropriate right-click menu option to approximate it. Note that Maple uses a special symbol e to represent the base of natural logarithms, whereas the letter e has no special significance.)

▶ *Solution*

(b) Enter and evaluate the definite integral $\int_{0}^{\infty} e^{-x^2} \ln(x)\, dx$, then approximate the result numerically.

▶ *Solution*

(c) Enter and evaluate the definite integral $\int_{0}^{y} e^{-x^2}\, dx$, then evaluate it at $y = 1$ and approximate the result numerically. (Hint: use the appropriate right-click menu options for the second and third steps.)

▶ *Solution*

Trigonometric functions

(a) Enter, evaluate and, if necessary, approximate each of the following expressions to obtain an **explicit** numerical value: $\cos(0)$, $\cos\left(\dfrac{\pi}{2}\right)$, $\sin\left(\dfrac{\pi}{4}\right)$. (Hint: use the appropriate right-click menu option to perform the numerical approximation.)

▶ *Solution*

(b) Enter, evaluate and then expand each of the following expressions: $\sin(\theta + \phi)$, $\sin(2\,\theta)$, $\sin(2\,\theta + \phi)$. (Hint: use the Greek palette to enter the Greek letters and use the appropriate right-click menu option to expand the expressions.)

▶ *Solution*

(c) (Trickier!) Enter, evaluate and then approximate the expression $\tan\left(\dfrac{\pi}{8}\right)$ to obtain an explicit numerical value. Compute an exact numerical value for $\tan\left(\dfrac{\pi}{8}\right)$ as follows. Construct an equation of the form $\tan(2\,\theta) = expanded\ value$, solve it for $\tan(\theta)$, and evaluate the resulting equation at $\theta = \dfrac{\pi}{8}$. Approximate this exact equation to check its validity (and note the numerical error). (Hint: use the right-click menu options as appropriate. To enter π into a dialogue box, type it as Pi.)

▶ *Solution*

Limits and integrals

(a) The logarithm function is singular at the origin, but is this singularity integrable? To investigate this, enter and evaluate $\ln(0)$, enter and evaluate $\lim_{x \to 0} \ln(x)$, and enter and evaluate the indefinite integral $\int \ln(x)\,dx$. Then insert an empty two-dimensional plot using the *Insert* menu and select and drag the integrand ($\ln(x)$) onto the plot (whilst holding down the *Control* key so that you copy rather than move it). Note the colour of the graph and note the singularity. Why has the negative *x*-axis disappeared? Now select and drag the result of evaluating the integral onto the plot (which will copy automatically since Maple output is not editable). The resulting graph shows that the singularity is integrable.

► *Solution*

(b) There is a general results that "any power beats a logarithm", i.e. $\lim_{x \to 0} x^p \ln(x) = 0$ and $\lim_{x \to \infty} x^{-p} \ln(x) = 0$ for any $p > 0$. Investigate this claim using a few explicit values for p.

► *Solution*

Complex numbers and expressions

In the Common Symbols palette, there is a choice of three symbols (i, j, I) that can all be used to input the imaginary unit, as you can see by hovering the mouse pointer over them, but by default Maple will always use I as the imaginary unit in output (because i and j also have other conventional uses). However, you can instruct Maple to use almost any symbol, such as i, for both input and output of the imaginary unit by executing $interface(imaginaryunit = i)$.

(a) Convince yourself that Maple understands complex arithmetic. For example, enter and evaluate the complex numerical expressions $(1 + 2i)\,(3 + 4i)$ and $\dfrac{1 + 2i}{3 + 4i}$. (Don't forget to insert a space between the parentheses to imply multiplication or use more explicit multiplication!) Then use the right-click context menu to find the real and imaginary parts, complex conjugate, and modulus and argument of each number.

(b) Enter and evaluate the complex expression $\sin(x + i\,y)$ and then expand it.

► *Solution*

(c) De Moivre's formula can be written as $\cos(n\,\theta) + i\sin(n\,\theta) = \left(\cos(\theta) + i\sin(\theta)\right)^n$. (You can look it up in the Maple help system.) This formula provides a way to construct trigonometric identities for multiple arguments. Enter and evaluate De Moivre's formula, then evaluate it at (say) $n = 3$. Select the right-hand side and use the pop-up "clickable math" dialogue to expand only the right-hand side. Then take the real part of the resulting equation by applying the $\Re$ function available in the Common Symbols palette (which you can also type as Re). You will see that this does not simplify by default because Maple assumes that θ is a complex variable. Use the right-click menu to simplify the equation "assuming real". Do likewise for the imaginary part by applying the $\Im$ function available in the Common Symbols palette (which you can also type as Im).

► *Solution*

Sets and subsets

(a) Confirm that $\{3, 4, 5\}$ is a subset of $\{3, 4, 5, 6, 7\}$ but $\{1, 2, 3, 4, 5\}$ is not a subset of $\{3, 4, 5, 6, 7\}$. (Hint: use the subset operator in the Common Symbols palette.)

▶ *Solution*

(b) Perform the same subset test by using only set union or intersection and an equality test. (Hint: use the operators in the Common Symbols palette and use the right-click context menu to apply either of the commands `evalb` or `is` to get a true/false result.)

▶ *Solution*

Sequences and sets

(a) Use the Maple function `seq` to generate the sequence $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \dfrac{5}{6}, \dfrac{6}{7}$. (Hint: use $seq(f(i), i = a\,..\,b)$ where $f(i)$ is an appropriate expression in i and a and b are appropriate integer constants.)

▶ *Solution*

(b) Use the Maple function `seq` as above to generate the set $\left\{\dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}, \dfrac{1}{6}, \dfrac{1}{7}\right\}$ and assign it to the variable S. (Hint: a set is just a sequence inside braces.) Then use `seq` again but with the relation $\in$ instead of $=$ and the set S instead of an integer range to generate the set $\{x^{1/3}, x^{1/4}, x^{1/5}, x^{1/6}, x^{1/7}\}$ from the set S.

▶ *Solution*

Power set

(a) Look up the definition and the appropriate Maple function in the Maple help. Hence, compute the power set of a **small** set. The power set of a set S is usually written as either $P(S)$ or 2^S, because $\left|2^S\right| = 2^{|S|}$, hence the name. Here $|S|$ denotes the cardinality of the set S, which in Maple can be found using the function `nops`. Check the cardinality relation for the power set you have constructed.

▶ *Solution*

(b) Why is the cardinality of a power set as specified above? Consider how many subsets of each cardinality a set has and add these numbers (symbolically, using Maple). Use the binomial theorem to prove this result (again using Maple).

▶ *Solution*