Label quantities in trees and mappings

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Abstract

We consider labelled trees and $n$-mappings, i.e., functions from the finite set $[n] := \{1, 2, \ldots, n\}$ into itself, and study several problems related to the distribution of the labels along the paths in the tree or in the iteration orbit $(i, f(i), f^2(i), \ldots)$ of elements in a mapping, respectively. In particular we consider the occurrence and avoidance of certain label-patterns as well as natural generalizations of the notion of parking functions to trees and mappings.

For example, we study the exact and/or asymptotic behaviour of the number of records, runs, local minima and inversions in trees and mappings and enumerate alternating $n$-mappings, i.e., functions, where each element satisfies either $i < f(i) > f^2(i) < f^3(i) > \cdots$ or $i > f(i) < f^2(i) > f^3(i) < \cdots$. Moreover, we give enumerative results for the total number of parking functions for $m$ drivers and $n$ parking spaces forming either a size-$n$ tree or an $n$-mapping, respectively, i.e., pairs of a tree (or a mapping) and a sequence of preferred parking spaces such that all drivers are successful when following the allowed direction in case of conflict.

An analytic combinatorics treatment of most of the problems mentioned above naturally yields a study of certain linear and quasi-linear PDEs.