On the probability of planarity of a random graph near the critical point

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Abstract

Let $G(n, M)$ be the uniform random graph with $n$ vertices and $M$ edges. As is well-known $G(n, M)$ undergoes a sudden transition near the critical value $M = n/2$. Erdős and Rényi conjectured that the limiting probability

$$\lim_{n \to \infty} \Pr\{G(n, \frac{n}{2}) \text{ is planar}\}$$

exists and is a constant strictly between 0 and 1. Luczak, Pittel and Wierman (1994) proved this conjecture and Janson, Luczak, Knuth and Pittel (1993) gave lower and upper bounds for this probability.

Using generating functions and singularity analysis we determine the exact limiting probability of a random graph being planar near the critical point. More precisely, we compute the probability of planarity in the critical window $M = \frac{n}{2}(1 + \lambda n^{-1/3})$, $\lambda \in \mathbb{R}$. We extend these results to classes of graphs closed under taking minors, including series-parallel and outerplanar graphs. (Joint work with Vlady Ravelomanana and Juanjo Ru).