Khintchine-Meinardus method for asymptotic enumeration: Achievements and Limitations

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Abstract

Our objective in this talk is derivation of the asymptotics of the number \( c_n, \ n \to \infty \) of decomposable structures arising in statistical mechanics, combinatorics and quantum field theory. The method in the title is a combination of Khintchine’s probabilistic representation (1950) for \( c_n \) and the Meinardus analytical approach (1954) for the asymptotics of the generating function for the sequence \( \{c_n\} \). We extend the setting of the method to general multiplicative generating functions of \( \{c_n\} \) and derive an asymptotic formula for \( \{c_n\} \) when the Dirichlet generating function for the associated weights \( \{b_n\} \) has multiple simple poles on the positive real axis. Our asymptotic formula for the number of weighted partitions disproves the accepted belief in the physics literature that the main term in the asymptotics of \( c_n \) is determined by the rightmost pole. We also demonstrate the failure of the Khintchine-Meinardus method for models for which the normal local limit theorem does not hold. The main part of the talk is based on a joint paper with Dudley Stark, while the rest part is based on work in progress.

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