

TIME SERIES
Exercise Sheet 6

Please try to attempt all of the questions **before** the end of term. The feedback exercise is Question 1. You should post your solutions in the red box on the second floor of the Mathematics Building by **11.00 on Monday 9 January 2012**. These will be marked and returned to you the following week.

1. Classify each of the following models:

(a) $X_t = 0.7X_{t-12} + Z_t + 0.5Z_{t-12}$;

(b) $X_t = Z_t - 1.3Z_{t-4} + 0.4Z_{t-8}$;

(c) $X_t = 0.8X_{t-1} - 0.15X_{t-2} + Z_t - 0.3Z_{t-1}$;

(d) $(1 - 1.3B + 0.5B^2)(1 - B^{12})X_t = (1 + 0.5B)Z_t$;

(e) $(1 - 0.8B + 0.25B^2)\nabla X_t = Z_t$.

2. Using the method of matching coefficients, show that the autocorrelation function of the seasonal AR(1)₄ model given by

$$X_t = \Phi X_{t-4} + Z_t,$$

where $\{Z_t\} \sim WN(0, \sigma^2)$ and $|\Phi| < 1$, is

$$\rho(\tau) = \begin{cases} 1 & \text{if } \tau = 0, \\ \Phi^k & \text{if } \tau = 4k, k = 1, 2, \dots, \\ 0 & \text{otherwise.} \end{cases}$$

3. Give the form of the seasonal ARIMA(1, 1, 1) \times (0, 1, 1)₁₂ model.