Queen Mary
University of London

B. Sc. Examination by course unit 2011

MTH 6139 Time Series

Duration: 2 hours

Date and time: 13 May 2011, 2.30-4.30

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators ARE permitted in this examination. The unauthorized use of material stored in pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

Statistical functions provided by the calculator may be used provided that you state clearly where you have used them.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): D. S. Coad

Question 1 (17 marks) Let $\{X_t\}_{t=1,2,...}$ be a time series such that

$$X_t = m_t + s_t + Y_t,$$

where m_t denotes a polynomial trend of degree k, s_t denotes a seasonal effect with period length d and Y_t denotes a zero-mean stationary process with autocovariance function $\gamma_Y(\tau)$, $\tau = 0, \pm1, \pm2, \ldots$. It is assumed that $s_t = s_{t-d}$.

(a) Define the operators ∇ and ∇_d, and explain how they can be used to remove the trend and seasonality from the time series $\{X_t\}$. [6]

(b) Show that $\nabla_d X_t$ is a stationary process for $k = 1$ and give its autocovariance function. [5]

(c) Outline the main steps of the classical decomposition method for estimating the trend and seasonal effects. [6]

Question 2 (22 marks) Consider an MA(2) process of the form

$$X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2},$$

where $\{Z_t\} \sim WN(0, \sigma^2)$.

(a) Show that the autocorrelation function of this process is given by

$$\rho(\tau) = \begin{cases} 1 & \text{if} \quad \tau = 0, \\ \frac{\theta_1(1+\theta_2)}{1+\theta_1^2+\theta_2^2} & \text{if} \quad \tau = \pm1, \\ \frac{\theta_2}{1+\theta_1^2+\theta_2^2} & \text{if} \quad \tau = \pm2, \\ 0 & \text{if} \quad |\tau| > 2. \end{cases}$$

How does this function behave for an MA(q) process? [12]

(b) State a necessary and sufficient condition for the above MA(2) process to be invertible. For what values of θ_1 and θ_2 is the process invertible? [6]

(c) Define the seasonal MA$(2)_h$ process and give the equivalent condition for this process to be invertible. [4]

Question 3 (24 marks) Let the causal process for a time series $\{X_t\}$ be given by

$$X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2} = Z_t + \theta Z_{t-1},$$

where $\{Z_t\} \sim WN(0, \sigma^2)$.

(a) Write down the operator form of this process. Under what conditions would this be an ARMA$(2, 1)$ process? [4]

(b) Obtain the linear process form of this time series when $\phi_1 = 0.3$, $\phi_2 = 0.4$ and $\theta = 0.9$. [15]

(c) State the difference equations in terms of the autocorrelation function for an ARMA$(2, 1)$ process. How does this function behave for this process? [5]

Question 4 (20 marks) Consider an AR(1) process of the form

$$X_t = \phi X_{t-1} + Z_t,$$

where $\{Z_t\} \sim WN(0, \sigma^2)$.

(a) Describe the stationary and causal properties of this process for different values of ϕ. [6]

(b) Show that the autocorrelation function of a causal AR(1) process is $\rho(\tau) = \phi^{|\tau|}$ for $\tau = 0, \pm 1, \pm 2, \ldots$. What is its partial autocorrelation function? [8]

(c) Give the best linear predictor of X_{n+1} based on $X_1, \ldots, X_n$. Explain how you would estimate this predictor and give an approximate 95% prediction interval. [6]

Question 5 (17 marks) A time series data set, $\{x_t\}_{t=1,\ldots,240}$, was modelled as an ARIMA(p, d, q) process. Calculating ∇x_t produced a new series which resembled a stationary process. Given below are the first six values of the sample autocorrelation and partial autocorrelation functions of this new series.

Lag	ACF	PACF
1	-0.2946	-0.2946
2	-0.0868	-0.1901
3	-0.0698	-0.1775
4	0.0869	-0.0165
5	0.0624	0.0673
6	-0.0952	-0.0482

(a) Determine values of p, d and q which would best model the data. Give reasons for your answers. [8]

(b) Write down the suggested model for $\{X_t\}$, explaining your notation. [3]

(c) Having fitted the above model to the data, what residual diagnostics should be looked at, and why? [6]

End of Paper