MTH737U/MTHM737 Fluid Dynamics
Midterm Test

Duration: 45 min

Date and time: 29 Feb 2011, 10:10

Consider a viscous, incompressible fluid between two plates, one at $y = 0$ and the other at $y = h$. The upper plate is suddenly moved in the x-direction with speed U while the lower plate is held fixed. The resulting flow is of the form $\mathbf{u} = (u, 0, 0)$ and is governed by the equation,

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}\,, \tag{1}$$

where $u = u(y, t)$ and ν is the constant kinematic viscosity. Assume the fluid extends to $\pm\infty$ in the x-direction, without a pressure gradient in this direction.

(a) Show explicitly how equation (1) derives from the Navier-Stokes equation for an incompressible fluid. [20 marks]

(b) Sketch the situation and identify the initial and boundary conditions for u. [20 marks]

(c) Let $u(y, t) = u_0(y) + u_1(y, t)$, where u_0 is the steady state solution and u_1 is the solution to equation (1) with homogeneous boundary conditions. Obtain the steady state solution u_0. [20 marks]

(d) What is the appropriate initial condition for u_1?

[20 marks]

(e) Use separation of variables to obtain the solution,

$$u_1(y, t) \;=\; \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi y}{h}\right) \exp\left\{-\frac{n^2\pi^2\nu t}{h^2}\right\}\,,$$

which ultimately leads to the following full solution:

$$u(y, t) \;=\; U\left[\left(\frac{y}{h}\right) - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi y}{h}\right)\exp\left\{-\frac{n^2\pi^2\nu t}{h^2}\right\}\right].$$

[N.B., *you do not need to obtain this full solution.*] [20 marks]

 END OF TEST