Queen Mary, University of London

MIDTERM EXAMINATION

MTH6119 Fluid Dynamics

13 Feb 2009, 10.10 - 10.55 (45 minutes)

This paper contains 1 question. Answer it clearly and legibly, crossing out any false starts.

Calculators are NOT permitted in this examination.

DO NOT TURN PAGE UNTIL INSTRUCTED TO DO SO.

1. A viscous, incompressible fluid lies at rest in the region $0 < y < \infty$ and $-\infty < x < \infty$, above a rigid boundary at $y = 0$. At $t = 0$, the boundary is suddenly given an impulse in the positive x-direction with constant speed U.

(a) Assume the flow is of the form, $\mathbf{u} = [u(y,t), 0, 0]$, and is driven only by the motion of the boundary (hence $\partial p/\partial x = 0$). Show that the velocity $u(y,t)$ satisfies the equation,

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2},$$

where ν is the constant kinematic viscosity. [10 Marks]

(b) Specify the initial and boundary conditions in this problem. [15 Marks]

(c) Let $u = f(\eta)$, where $\eta = y/(\nu t)^{\frac{1}{2}}$, and derive the following equation for f:

$$f'' + \frac{1}{2}\eta f' = 0,$$

where the prime denotes the derivative with respect to the argument. [30 Marks]

(d) Obtain the general solution for f. Then, using the initial and boundary conditions, obtain

$$u(\eta) = U\left[1 - \frac{1}{\sqrt{\pi}}\int_0^\eta e^{-s^2/4}\,\mathrm{d}s\right],$$

where s is a dummy variable. [Hint: $\int_0^\infty \exp\{-s^2/4\}\,\mathrm{d}s = \sqrt{\pi}$] [30 Marks]

(e) From the solution, obtain the vorticity $\omega(y,t)$. Sketch ω as a function of y, at some time $t > 0$, and identify the characteristic, e-folding lengthscale. [15 Marks]

[End of examination paper.]