

MTH5106 Dynamics of Physical Systems
Midterm Revision Notes

1. INTRODUCTION

- (a) Space, time and speed scales
 - i. Acquiring a “feeling for” scales (order of magnitude)
 - ii. Some useful numbers:
 $R_e = 6.4 \times 10^6$ m, $1 \text{ yr} = 3.17 \times 10^7$ s, $c = 3 \times 10^8$ m s⁻¹, $g = 9.81$ m s⁻²,
 $G = 6.67 \times 10^{-11}$ N m² kg⁻²
- (b) Vectors:
 - i. Scalar (dot) product, vector (cross) product; algebraic and geometric views
 - ii. Displacement, \mathbf{r} ; velocity, $\mathbf{v} \equiv d\mathbf{r}/dt \equiv \dot{\mathbf{r}}$; acceleration, $\mathbf{a} \equiv d\mathbf{v}/dt \equiv d^2\mathbf{r}/dt^2 \equiv \ddot{\mathbf{r}}$; be able to go “forwards and backwards” (i.e., differentiation and integration) using the definitions
- (c) circular motion, Taylors expansion and approximations
 - i. Sketch trajectory, velocity and acceleration; velocity along and acceleration normal (inward) to the path
 - ii. Using expansion to approximate numbers and functions with successive accuracy

2. NEWTONS LAWS

- (a) Units and dimensional balance: checking units and balancing dimensions
- (b) Forces; gravitational attraction
 - i. Newton I, II and III
 - ii. Newton’s universal gravitational law: mathematical structure and comparison with Coulomb’s law

3. MOMENTUM AND KINETIC ENERGY

- (a) Momentum, $\mathbf{p} = m\mathbf{v}$, and its conservation: $m_1\mathbf{v}_1^i + m_2\mathbf{v}_2^i = m_1\mathbf{v}_1^f + m_2\mathbf{v}_2^f$.
- (b) Kinetic energy, $\frac{1}{2}m\mathbf{v}^2$, and its conservation: $\frac{1}{2}m_1(\mathbf{v}_1^i)^2 + \frac{1}{2}m_2(\mathbf{v}_2^i)^2 = \frac{1}{2}m_1(\mathbf{v}_1^f)^2 + \frac{1}{2}m_2(\mathbf{v}_2^f)^2$.
- (c) Two-particle collision: elastic collision, relative velocity

4. WORK

- (a) Work, $W = \int \mathbf{F} \cdot d\mathbf{r}$
- (b) Integrating the equation of motion:
$$\mathbf{F} \cdot \mathbf{v} = m \frac{d\mathbf{v}}{dt} \cdot \mathbf{v} \Rightarrow d(\mathbf{F} \cdot \mathbf{r}) = d\left(\frac{1}{2}m\mathbf{v}^2\right), \text{ if } \mathbf{F} = \mathbf{F}(\mathbf{r})$$
- (c) Work-energy theorem: $\Delta(\text{WORK}) = \Delta(\text{KE})$

5. POTENTIAL ENERGY, U

- (a) PE, $U = mgz$, in one dimension under gravity
- (b) Conservation of energy: Total Energy, $E = KE + PE$
- (c) Given a one-dimensional potential energy function $U = U(x)$, sketch and find local stable and unstable points: making use of dU/dx and d^2U/dx^2 .

6. GRADIENT, ∇ , a differential operator which transforms like a vector

- (a) Given scalar function (field) ϕ and vector function (field) \mathbf{A} :
 - i. gradient, $\nabla\phi$; divergence, $\nabla \cdot \mathbf{A}$; curl, $\nabla \times \mathbf{A}$; Laplacian, $\nabla^2\phi$ and $\nabla^2\mathbf{A}$
- (b) Force from potential energy: $\mathbf{F} = -\nabla U$
- (c) Conservative force $\mathbf{F}_c \implies \nabla \times \mathbf{F}_c = 0$ and Stokes theorem,

$$\oint_c \mathbf{F} \cdot d\mathbf{r} = \int_S \nabla \times \mathbf{F} \cdot d\mathbf{a},$$

where the circuit c encloses the surface S . In a conservative force field, the work done between two points is path-independent.

**** SOME GENERAL TIPS ****

- Study lecture notes, go over coursework, then do practice problems, including old exams.
- Practise what you've learnt, or have been shown, in lectures and tutorials (as well as courseworks). Chances are they will come up again, somewhere, unless explicitly told otherwise
- DO NOT TRY TO MEMORISE "FORMULAE" OR RELY ON "GUESSING" EXAM QUESTIONS (especially if you've not been attending lectures and/or skimming out on coursework problems); these are PROVED bad strategies.
- Revise now, revise frequently and (very important!) revise afterwards.
- On the exam day, take a minute or two to read through the entire exam BEFORE you start; do the easy questions first (to rack up marks).
- Get a good night's rest the day before and have fun with the exam; remember, it's only a small part of your overall mark!