MTH5106 – Dynamics of Physical Systems **Dr. J. Cho**

Problem Set 4 (4 questions) **Due Tuesday, 3 Nov 2009, at 12:00 in the Lec Room**

Your solutions should be handed in *before* start of the lecture on the due date. Late scripts will *not* be accepted without proper justification. **This set consists of two pages.**

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1. Do the following:

   (i) Usng Newton's second law, show that, for an object of mass m, the change of kinetic energy in going from point 1 to point 2 is equal to the total work done on the object going from point 1 to point 2.

   (ii) An object of mass m, initially at rest at point 1, moves with a velocity $\mathbf{v}$ to point 2 in a constant gravitational force field ($\mathbf{F} = -mg\hat{\mathbf{z}}$). Show that the change in its kinetic energy is the the work done by the gravitational field on the object, $-mg\Delta z = -mg(z_2 - z_1)$. [Hint: $\mathrm{d}\mathbf{r} = \mathrm{d}x\,\hat{\mathbf{x}} + \mathrm{d}y\,\hat{\mathbf{y}} + \mathrm{d}z\,\hat{\mathbf{z}}$.]

   (iii) Draw a square circuit of side 1 in the x-z plane such that the lower left corner is at the origin $(0,0,0)$ and the upper right corner is at the location $(1,0,1)$. Starting from the origin, take a complete circuit counter-clockwise around the square loop, back to the origin, evaluating the integral, $I = \oint \mathbf{F}\cdot \mathrm{d}\mathbf{r} = \oint(-mg\hat{\mathbf{z}})\cdot \mathrm{d}\mathbf{r}$. Show that $I = 0$.

2. At $t = 0$, a stone of mass m is cast off a ledge with a horizontal velocity v_0 (the vertical velocity is zero). Gravity points straight down and symmetry is assumed in the $\hat{\mathbf{z}}$-direction (i.e., motion confined to the x-y plane). There is frictional force, which is of the form: $-\alpha m(v_x^2, v_y^2)$, where $\alpha > 0$ is a constant.

   (i) What is the unit (in SI) of α?

   (ii) Orient the coordinate axis so that $\hat{\mathbf{x}}$ points to the right and $\hat{\mathbf{y}}$ points downward. Given the velocity, $\mathbf{v}(t) = (v_x(t), v_y(t))$, write down the horizontal equation of motion – a differential equation for the velocity v_x – and then solve it, subject to the initial condition.

   (iii) Write down the vertical equation of motion – a differential equation for v_y – and then solve it, subject to the initial condition. [Hint:

   $$\int \frac{1}{b - a\xi^2}\,\mathrm{d}\xi = \frac{1}{\sqrt{ab}} \tanh^{-1}\left(\xi\sqrt{\frac{a}{b}}\right),$$

   where a and b are positive constants. N.B., the initial condition for v_y will be automatically satisfied when this result is used; check this after obtaining $v_y(t)$.]

   (iv) Analyse what happens at early times – i.e., $t \to 0$ (but not $= 0$) such that $\sqrt{\alpha g}\,t \ll 1$ – and at late times, $t \to \infty$. For the latter, imagine that there is plenty of room for the falling stone.

[continued on the next page..]

3. A (one-dimensional) potential function is given by the following:

$$U(x) = x^3 - 3x^2 - x + 3.$$

(i) Sketch the function.

(ii) Find all the equilibrium points.

(iii) Determine whether each of the equibrium points is stable or unstable.

(iv) Copy your sketch of $U(x)$ from part (i). Indicate on the copy the direction of forces around each of the equilibrium points.

4. Do the following:

(i) Briefly, in your own words, state Stokes' theorem. Use a simple sketch/diagram to aid your statement.

(ii) Briefly, describe three ways one can assess whether a given force field is conservative.

(iii) Show that the field,

$$\mathbf{F}(\mathbf{r}) = -\frac{k}{r^2}\hat{\mathbf{r}},$$

where $k > 0$ is a constant and $\mathbf{r} = r\hat{\mathbf{r}} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$, is conservative.