MTH5106 – Dynamics of Physical Systems **Dr. J. Cho**

Problem Set 3 (4 questions) **Due Tuesday, 27 Oct 2009, at 12:00 in the Lec Room**

Your solutions should be handed in *before* start of the lecture on the due date. Late scripts will *not* be accepted without proper justification. **This set consists of two pages.**

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1. A rock is thrown upward with velocity $\mathbf{v}_0$, with an angle θ from the horizontal ground. You may neglect friction in this problem. Gravity $\mathbf{g}$ points straight downward, and the rock is thrown in the positive $\hat{\mathbf{x}}$-direction.

   (i) Sketch the situation.

   (ii) What is the vertical and horizontal components of the velocity, $\mathbf{v}^{\text{top}} = (v_x^{\text{top}}, v_y^{\text{top}})$, at the highest point?

   (iii) What is the vertical component of the velocity v_y^{grd}, when the rock hits the ground?

   (iv) What is time t_u for the rock to reach the highest point in its trajectory? Show that the time t_d the rock takes from the highest point back to the ground, on its way down, is same as t_d.

2. Consider the situation in Question 1.

   (i) What is the horizontal distance X from the point at which the rock was thrown in terms of v_0, g and θ?

   (ii) Show that there is an optimal angle θ_{max} which leads to the maximum, $X = X_{\text{max}}$, given v_0 and g.

   (iii) Briefly describe how, in principle, one could measure the value of g by repeatedly throwing the rock with a known v_0 and θ (say at θ_{max}) and measuring X_{max}. In practice, you would want to use a machine which can always throw at the same speed and angle without getting tired.

3. In your own words, briefly state the following:

   (i) Newton's first law.

   (ii) Newton's second law.

   (iii) Newton's third law.

   (iv) Newton's gravitational law.

   Be sure to define the variables, if you use them.

4. [N.B., this problem will *not* be chosen for marking.] Consider again the situation described in Question 1 above. However, this time the ground is not level (i.e., horizontal); it slopes downward from the horizontal by an angle, $\phi < \pi/2$, so that the angle from the ground to the $\hat{\mathbf{v}}_0$-direction is $\psi = \phi + \theta$.

   (i) Sketch the situation.

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(ii) Now, rotate the coordinate axis so that the $\hat{\mathbf{x}}$-direction is along the sloping ground. In this rotated coordinate system, show that $\mathbf{g}$ decomposes to give (constant) acceleration in both horizontal and vertical directions (unlike in the no-slope case). Sketch the new situation with respect to the rotated coordinates.

(iii) Making sure that all the signs are consistent and making use of ψ, show that the horizontal distance L (i.e., the distance along the slope) is:

$$L = \frac{2v_0^2 \sin\psi}{g\cos\phi}(\cos\psi + 2\tan\phi\sin\psi).$$

(iv) Check the above expression by showing that it reduces appropriately to the no-slope case, when $\phi = 0$.