MTH5106, DPS CW3 Sol'n

J. Cho

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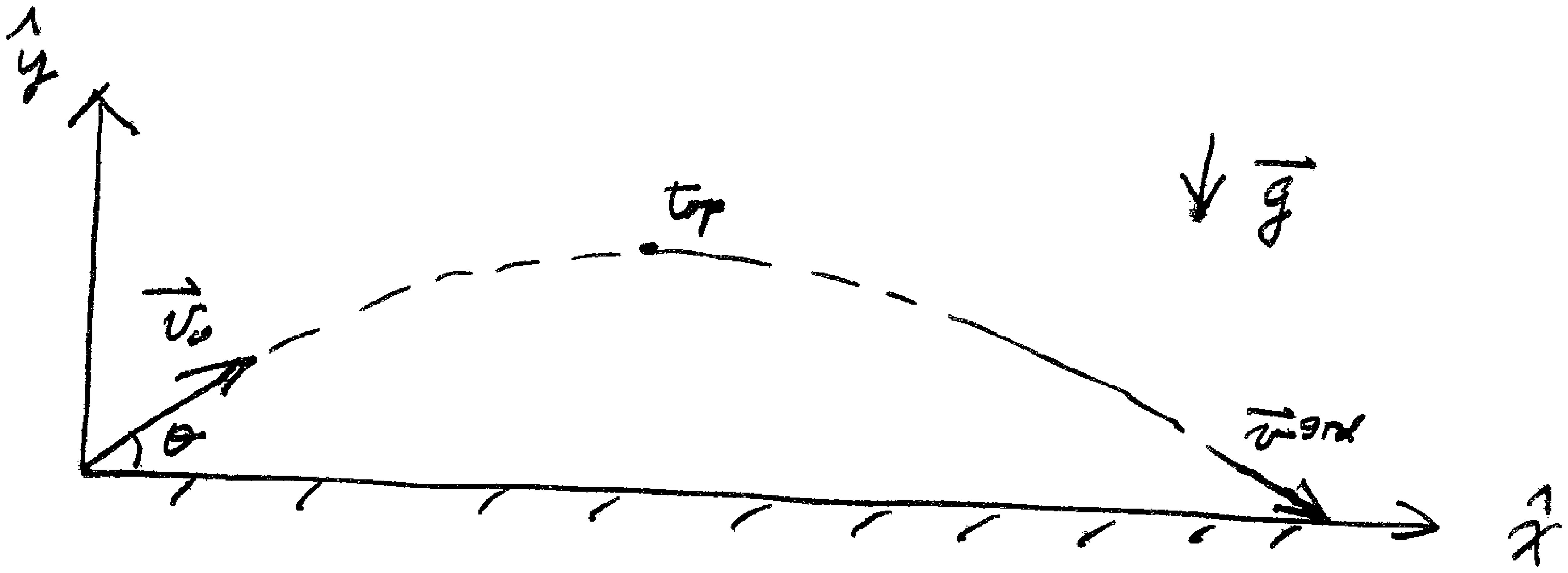
(i)



(ii) $v_x^{top} = v_0 \cos\theta$

$v_y^{top} = 0$

(iii) $\cancel{(v_y^{top})^2}^{\,0} = (v_{0y})^2 - 2g(y^{top} - \cancel{y^{grd}}^{\,0})$

$\Rightarrow \quad v_{0y} = \sqrt{2gy^{top}}$ (n.b., $\oplus$ branch taken; see below, where $\ominus$ branch is taken).

but, also,

$(v_y^{grd})^2 = \cancel{(v_y^{top})^2}^{\,0} - 2g(\cancel{y^{grd}}^{\,0} - y^{top})$

$\Rightarrow \quad v_y^{grd} = \sqrt{2gy^{top}}$

$\therefore \quad \boxed{v_y^{grd} = v_{0y} = v_0 \sin\theta}$ * The direction is $-\hat{y}$ *

$\therefore \boxed{v_y^{grd} = -v_0 \sin\theta}$

(iv) $\cancel{v_y^{top}}^{\,0} = v_{0y} - g t_u \Rightarrow \boxed{t_u = \dfrac{v_{0y}}{g} = \dfrac{v_0 \sin\theta}{g}}$

From part (iii), we have $v_y^{grd} = -v_{0y}$. Now, on the downward journey,

$v_y^{grd} = \cancel{v_y^{top}}^{\,0} - g t_d = -v_{0y} \Rightarrow \boxed{t_d = \dfrac{v_{0y}}{g} = t_u}$

$\boxtimes$.

$\boxed{2}$

(i)

$$t = t_u + t_d = 2t_u = \frac{2v_{0y}}{g}$$

$$X = \cancel{x_0}^{\,0} + v_{0x}t + \cancel{\tfrac{1}{2}a_x t^2}^{\,0} = \frac{2v_{0y}v_{0x}}{g} = \boxed{\frac{2v_0^2}{g}\sin\theta\cos\theta}$$

(ii) $$\frac{dX}{d\theta} = \frac{d}{d\theta}\left(\frac{v_0^2}{g}\sin 2\theta\right) = \frac{v_0^2}{g}\cdot 2\cos 2\theta = 0$$

$\Rightarrow X_{max}$ corresponds to when $\cos 2\theta_{max} = 0$

$$\Rightarrow \boxed{\theta_{max} = \frac{\pi}{4}, \text{ for } \theta < \frac{\pi}{2}}$$

(iii) From parts (i) and (ii), we see that

$$X_{max} = \frac{2v_0^2}{g}\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{v_0^2}{g}$$

$$\Rightarrow g = \frac{v_0^2}{X_{max}}.$$

$\therefore$ given v_0, by repeatedly measuring X_{max}, in principle, one could measure g.

$\boxed{3}$

(i) An object in uniform motion (or at rest) stays in uniform motion (or at rest), unless acted upon by an external force.

(ii) Force equals mass times acceleration (of the body induced by the force: $\vec{F} = m\vec{a}$).

(iii) For every "action" on an object, there is an equal and opposite "reaction" by the object.

(iv) Force $\vec{F}$ between two masses, m_1 and m_2, separated by a distance r is such that

$$\vec{F} = -G\frac{m_1 m_2}{r^2}\hat{r},$$ where the minus

sign represent attraction (as opposed to repulsion) and the force is directed along the line joining m_1 and m_2. G is a constant ($= 6.67\times10^{-11}\frac{m^3}{kg\,s^2}$).

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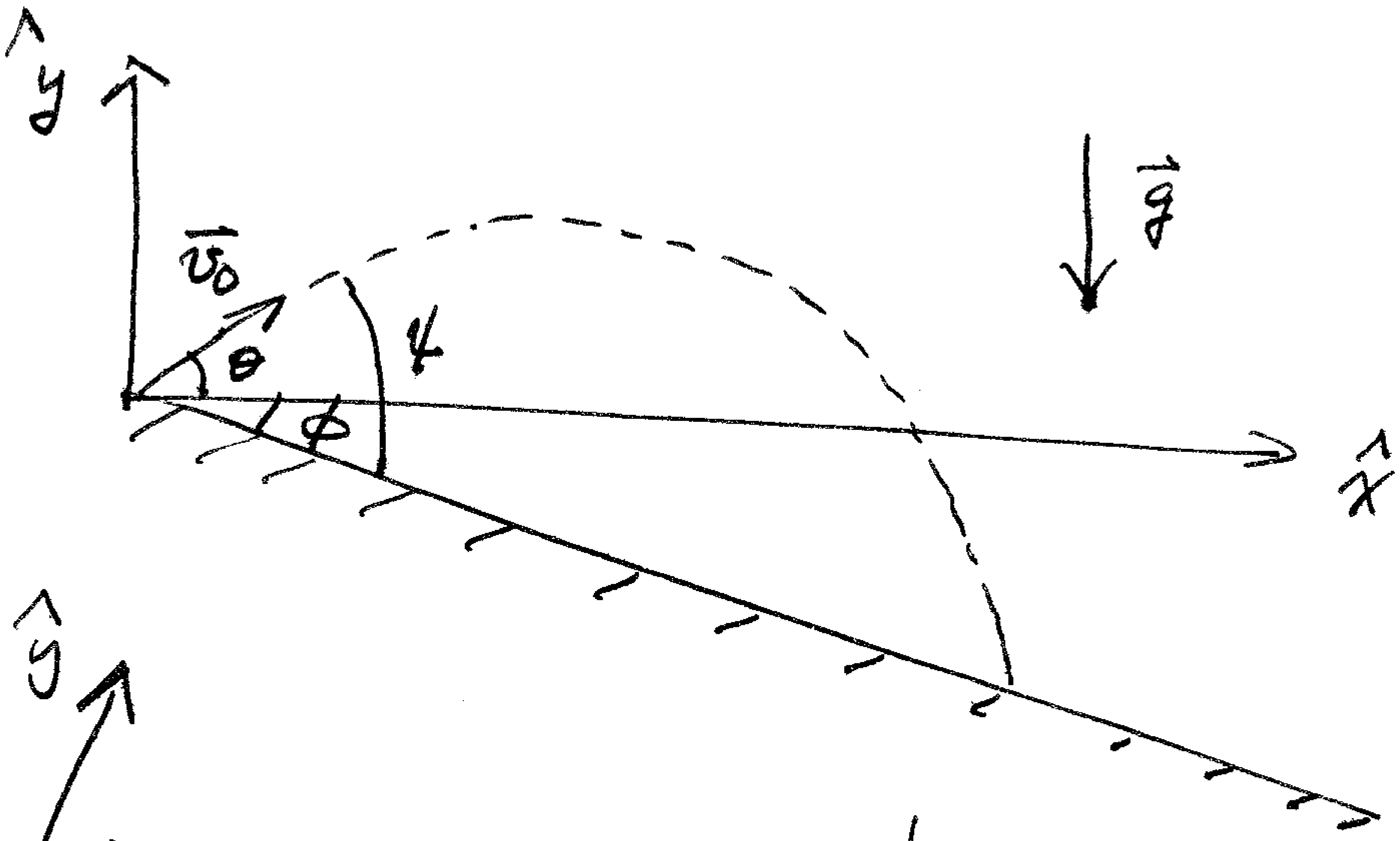
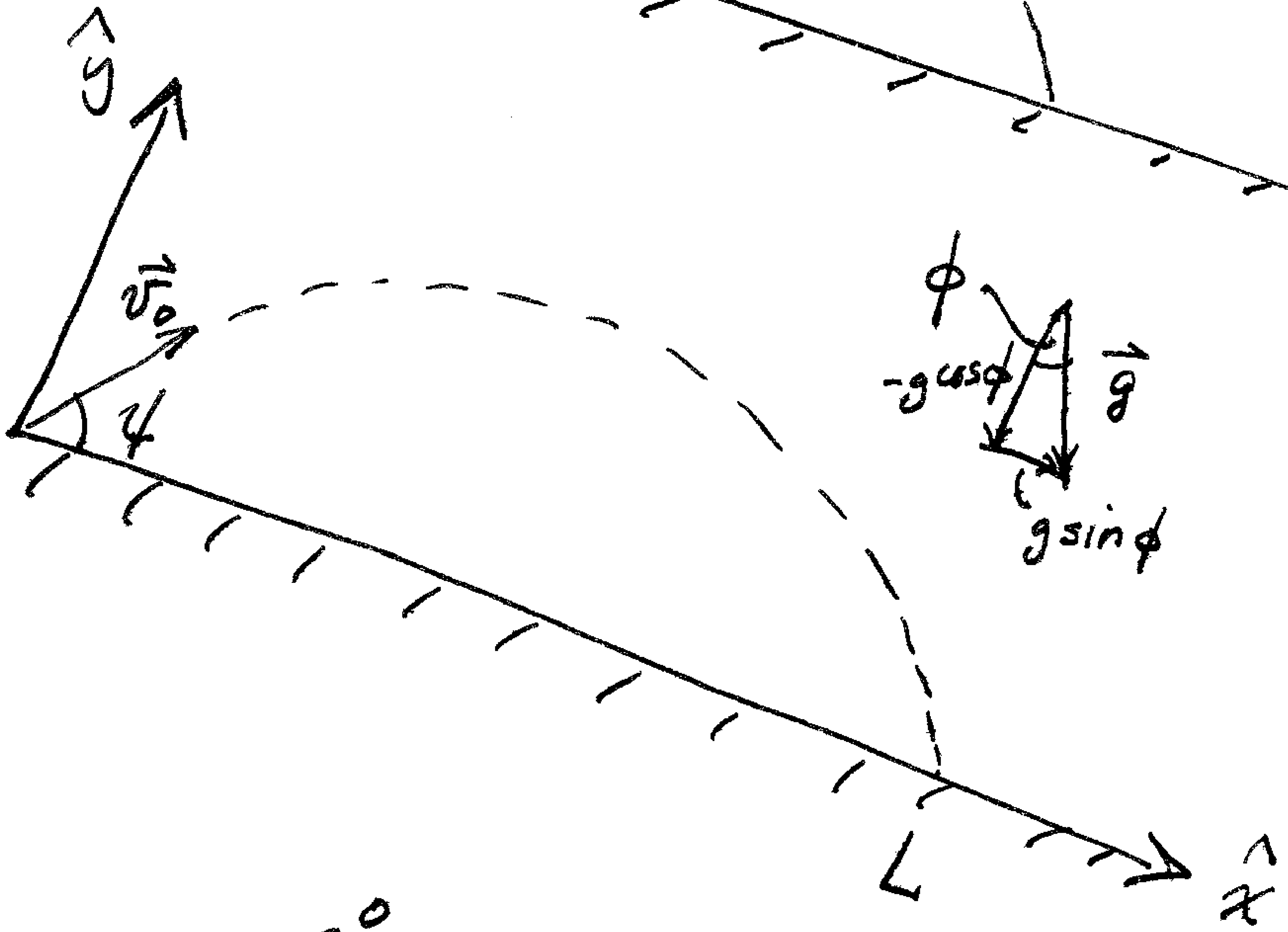
(i)



(ii)



(iii) $L = \cancel{x_0}^{\,0} + v_{0x} t + \frac{1}{2} a_x t^2$

$0 = 0 + v_{0y} t + \frac{1}{2} a_y t^2 \Rightarrow \boxed{-\frac{2 v_{0y}}{a_y} = t \quad ; \quad a_y = -g\cos\phi}$

t is the time of flight.

$\therefore \quad L = -\frac{2 v_{0y}}{a_y}\left(v_{0x} - \frac{2 a_x v_{0y}}{2 a_y}\right)$

$$\boxed{L = \frac{2 v_0^2 \sin\psi}{g\cos\phi}\left(\cos\psi + \tan\phi \sin\psi\right)}$$

n.b., dimensions work out

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(iv) when $\phi = 0$, $\psi = \theta$. Therefore L reduces to

$L = \frac{2 v_0^2}{g} \sin\theta \cos\theta$, as was obtained in Prob. 2.