MTH5106 – Dynamics of Physical Systems **Dr. J. Cho**

Problem Set 2 (4 questions) **<u>Due Friday, 20 Oct 2009, at 12:00 in the Lec Room</u>**

Your solutions should be handed in *before* start of the lecture on the due date.
Do NOT use a calculator for any of the questions. Numerical answers correct to within 10% will be accepted. You may assume $g \approx 10$ m s^{-2}.

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1. Given the two-dimensional position vector $\mathbf{r}(t) = a\sin(\omega t)\mathbf{i} + b\cos(\omega t)\mathbf{j}$, where a, b and ω are positive constants and $a > b$:

   (i) Sketch roughly the trajectory traced by $\mathbf{r}(t)$. Be sure to indicate the direction of the trajectory as t increases.

   (ii) Given $\mathbf{r}(t)$ above, compute the velocity $\mathbf{v}(t)$ and sketch $\mathbf{v}$. What is the speed $|\mathbf{v}|$?

   (iii) Using the definition of the time derivative of a vector function $\mathbf{q}(t)$,

   $$\frac{\mathrm{d}\mathbf{q}(t)}{\mathrm{d}t} = \lim_{\Delta t \to 0}\left[\frac{\mathbf{q}(t+\Delta t) - \mathbf{q}(t)}{\Delta t}\right],$$

   show that $\mathbf{v}(t)$ obtained in part (ii) is correct *geometrically* by subtracting $\mathbf{r}$ at two successive times, separated by a small time interval Δt.

2. Given a constant acceleration $\mathbf{a}$, initial velocity $\mathbf{v}_0$ and initial position $\mathbf{r}_0$, show that

   (i) $\mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a}t$

   (ii) $\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$

   (iii) $\mathbf{v}^2(t) = \mathbf{v}_0^2 + 2\mathbf{a}\cdot[\mathbf{r}(t) - \mathbf{r}_0]$

   [N.B., we have not defined “reciprocal vectors”. Hence, expressions like “$1/\mathbf{a}$” and “$t = (\mathbf{v} - \mathbf{v}_0)/\mathbf{a}$” cannot be used. In this case, one might try making use of the dotting operation – e.g., $(\mathbf{r} - \mathbf{r}_0)\cdot\mathbf{a}$.]

3. A Porsche sedan accelerates from rest so that its speedometre is reading 72 km h^{-1} after 5 seconds of travel. The acceleration is constant.

   (i) Compute the acceleration, in SI units, resulting in the speed at 8 seconds.

   (ii) Compute the distance traversed in this time. Compare this distance with that which would be obtained by a Mercedez sedan moving at a constant speed of 100 km hr^{-1}.

   (iii) From the 5 seconds mark, how long will it take for the Porsche to catch up to the Mercedez (and then pass it)?

4. A little girl is standing on the ground at $y = 0$ and throws a ball with speed, $v_0 = 5$ m s^{-1}, straight up into the air. Gravity points downward: $\mathbf{g} = -g\mathbf{j}$. So, eventually the ball stops moving upward.

   (i) At the point when the ball stops moving upward, how long has the ball been up in the air?

   (ii) At that point, what is the height y?

   (iii) Compute the quantity, $\frac{1}{2}v_0^2$, the kinetic energy per mass of the ball when it was thrown. Now, compute the quantity, gy, the potential energy per mass of the ball at the point where it stopped moving upwards. Compare the two quantities. How do they compare? How can this be shown directly from one of the expressions derived in Question 2?