MTH5106 – Dynamics of Physical Systems **Dr. J. Cho**
Problem Set 1 (5 questions) **Due by 10:00 Friday, 9 October 2009**

Your solutions should be handed in *before* start of the lecture on the due date.
Do NOT use a calculator for any of the questions. Numerical answers correct to within 10% will be accepted.

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1. Briefly, answer the following:

   (i) There are 60 seconds in a minute, 60 minutes in a hour, 24 hours in a day and 365 days in a year. How many seconds are there in a year?

   (ii) Why is the following statement obviously incorrect?
   The speed of light c is 2.99×10^8 $m\ s^{-2}$.

   (iii) Suppose you did not know that the way to calculate the distance x travelled by something moving at a constant speed v for some time t was "$x = vt$". How might you have guessed that this could be so from the units?

   (iv) Suppose acceleration a is defined as $\mathrm{d}^2x/\mathrm{d}t^2$, where $x = x(t)$. What are the units (in S.I.) of a?

2. The vectors, $\mathbf{a} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}$, are given. Do the following:

   (i) Compute $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$

   (ii) Show that $\mathbf{a} \cdot \mathbf{b}$ commutes, but $\mathbf{a} \times \mathbf{b}$ does not.

   (iii) Find a vector that is orthogonal to $\mathbf{b}$.

   (iv) Find a vector that is simultaneously orthogonal to $\mathbf{a}$ *and* $\mathbf{b}$.

3. Show the following, given vectors $\mathbf{a} = \mathbf{a}(t)$, $\mathbf{b} = \mathbf{b}(t)$ and $\mathbf{c} = \mathbf{c}(t)$:

   (i) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})$

   (ii) $\mathrm{d}(\mathbf{a} \cdot \mathbf{a})/\mathrm{d}t = 2\mathbf{a} \cdot \mathrm{d}\mathbf{a}/\mathrm{d}t$

4. Given the scalar function $y = y(x)$ and the vector function $\mathbf{a} = \mathbf{a}(t)$, do the following:

   (i) Write down the definition of $\mathrm{d}y/\mathrm{d}x$ in terms of the interval Δx in the limit as $\Delta x \to 0$ and the definition of $\mathrm{d}\mathbf{a}/\mathrm{d}t$ in terms of the interval Δt in the limit as $\Delta t \to 0$.

   (ii) Show that $y(x + \Delta x) = y(x) + y'(x)\Delta x$ and $\mathbf{a}(t + \Delta t) = \mathbf{a}(t) + \mathbf{a}'(t)\Delta t$ to first order in Δx and Δt, respectively, where the prime indicates differentiation with respect to the independent variable.

5. Use expansion to *estimate* the following:

   (i) 35^2

   (ii) $e^{0.1}$, where $e \approx 2.718$

   (iii) $\sqrt{\pi}$, where $\pi \approx 3.141$

   (iv) $\sin 6^\circ$ (n.b., $180^\circ = \pi$ radians)