MAS 226, 2007 MIDTERM SOLUTIONS

$\boxed{1}$ $\vec{A} = (-xy, 1, 0)$, $\vec{B} = (-y, y^2, z)$

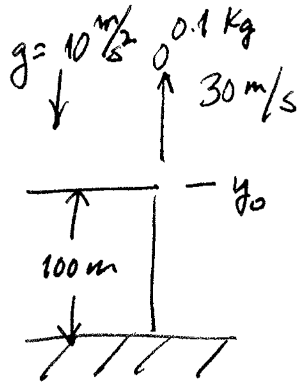
a) $\vec{A}\cdot\vec{B} = xy^2 + y^2 = \boxed{y^2(x+1)}$

b) $\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -xy & 1 & 0 \end{vmatrix} = (0, 0, x) = \boxed{x\hat{k}}$

c) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0$, since $\vec{\nabla} \perp (\vec{\nabla}\times\vec{B})$

or, $\vec{\nabla}\cdot\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & y^2 & z \end{vmatrix} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot(0, 0, 1) = \boxed{0}$

$\boxed{2}$



a) $0 = v_0 - gt \Rightarrow t = \frac{v_0}{g} \Rightarrow y - y_0 = v_0 t - \frac{1}{2}gt^2$

$\Rightarrow y - y_0 = \frac{v_0^2}{g} - \frac{1}{2}\cancel{g}\frac{v_0^2}{g\cancel{^2}} = \frac{v_0^2}{2g}$

$\Rightarrow y - y_0 = \frac{30^2}{2\cdot 10} = \frac{90\cancel{0}}{2\cancel{0}} = \boxed{45\,m}$

b) time to reach max. height from y_0 = time to reach y_0, dropping from y (which we can set to 0 without loss of generality):

hence, $\cancelto{0}{y} - y_0 = -\frac{1}{2}gt^2 \Rightarrow t_d = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{90}{10}} = 3\,s$. Therefore, the ball has been up for $\boxed{6\,s}$.

c) Since all of the kinetic energy comes from the initial potential energy through conversion, $mgh = \frac{1}{10}\cdot 10\cdot 145 = \boxed{145\,J}$

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a) A mass stays at rest or is in uniform motion unless acted upon by an external force.

b) Two masses, m_1 and m_2, are attracted along the line joining them by the force, $G\frac{m_1 m_2}{r^2}\hat{r}$, where r is the distance between the two masses, $\hat{r}$ the direction of the position vector between them and G is a constant.

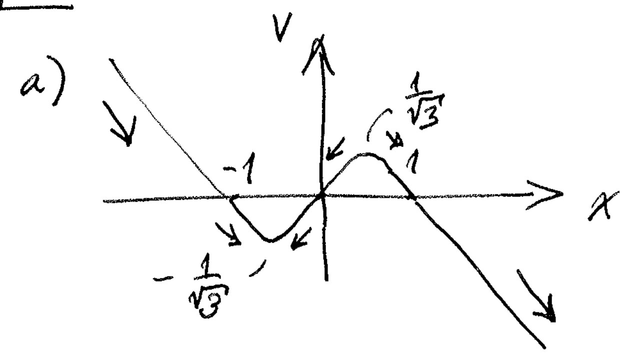
c). The gravitational law is purely an attractive force law, whereas Coulomb's law is both attractive <u>and</u> repulsive (depending on the sign of the charge).

→ other possible answers:

- the constants of proportionalities, G an K (in $\vec{F}_c = K\frac{q_1 q_2}{r^2}\hat{r}$) are different.
- the gravitational law acts on the masses, whereas Coulomb's law on charges

4 $V(x) = -x(x-1)(x+1)$

a)



equilibrium pts:

$$V = -x(x^2-1) = x - x^3$$

$$\frac{dV}{dx} = 1 - 3x^2 = 0 \Rightarrow 3x^2 = 1$$

$$\Rightarrow \boxed{x = \pm\frac{1}{\sqrt{3}}}$$

b) $\frac{d^2V}{dx^2} = -6x \Rightarrow \begin{cases} x = \frac{1}{\sqrt{3}} \Rightarrow \frac{d^2V}{dx^2} = \frac{-6}{\sqrt{3}} < 0 \Rightarrow \text{local max (unstable)} \\ x = -\frac{1}{\sqrt{3}} \Rightarrow \frac{d^2V}{dx^2} = \frac{6}{\sqrt{3}} > 0 \Rightarrow \text{local min (stable)} \end{cases}$

c) Use $\vec{F} = -\frac{dV}{dx}\hat{\imath}$. see sketch above.

$\boxed{5}$ $U(x,y,z) = -U_0\, e^{-(x^2+y^2+z^2)}$

a) $\vec{F} = -\vec{\nabla} U = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot -U_0\, e^{-(x^2+y^2+z^2)}$

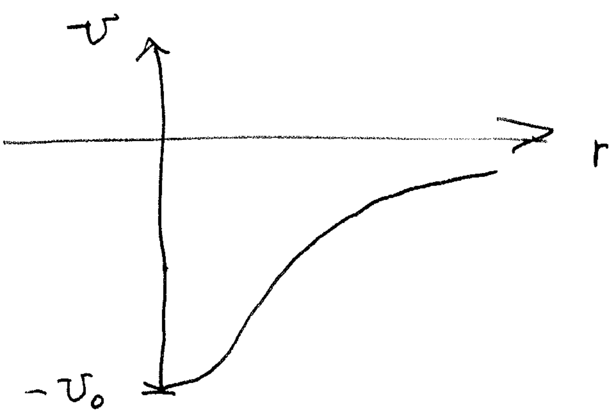
$$= U_0 \cdot -2\,(x, y, z) \cdot e^{-(x^2+y^2+z^2)}$$

$$= -2U_0\, e^{-(x^2+y^2+z^2)}(x,y,z) = \boxed{2U(x,y,z) = 2U\vec{r}}$$

b) $\vec{F}$ is conservative by construction, since

$\vec{\nabla} \times \vec{F} = -\vec{\nabla} \times \vec{\nabla} U = 0$

c)



To get out of the center, we need U_0 amount of energy.