# Queen Mary, University of London

## Midterm Examination

**MAS226, Dynamics of Physical Systems**

Friday, 10 November 2006, 15:10-16:00

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- Vectors are denoted by boldface type (e.g., $\mathbf{a}$, $\mathbf{A}$) while scalars are in italic (e.g., $a$, $A$).
- Calculators are *not* allowed.
- You may take gravity $g$ to be 10 m s$^{-2}$. Also, $\sin 30^\circ = \frac{1}{2}$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

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Attempt **ALL** of the following questions. There are **8** questions.

1. (i) Briefly (i.e., in one or two sentences), state the difference between a scalar field and a vector field.
   (ii) Give an example of a scalar field and a vector field. **[10 marks]**

2. Briefly explain how a gradient operator is like a vector. **[10 marks]**

3. Given three vectors, $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{b} = 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, compute:
   (i) $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$
   (ii) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ **[10 marks]**

4. Given two vectors, $\mathbf{a} = (3x^2 - 2y)\mathbf{i} + (1 - 2z)\mathbf{j} + (5y + 2z)\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + (3y^2 + 2z^2)\mathbf{j}$, and the gradient operator, $\nabla$, compute:
   (i) $\nabla(\nabla \cdot \mathbf{a})$
   (ii) $\nabla \cdot (\nabla \times \mathbf{b})$ **[10 marks]**

5. Given a constant acceleration $\mathbf{a}$, initial location $\mathbf{x}_0$, and initial velocity $\mathbf{v}_0$, derive the general expressions for:
   (i) velocity, $\mathbf{v} = \mathbf{v}(t)$, in terms of $\mathbf{a}$ and $\mathbf{v}_0$. You may take the initial time to be 0.
   (ii) location, $\mathbf{x} = \mathbf{x}(t)$, in terms of $\mathbf{a}$, $\mathbf{v}_0$, and $\mathbf{x}_0$. You may again take the initial time to be 0. **[10 marks]**

6. (i) An arbitrary force, $\mathbf{F}$, is conservative. How is the force expressed in terms of the potential (give a mathematical relationship)? Is such a potential unique (i.e., only one satisfying the relationship)? Explain.
   (ii) Now, consider the force field, $\mathbf{F} = 2\mathbf{i} + (3x - z^2)\mathbf{j} + (2y^2 - 3z)\mathbf{k}$. Is this field conservative? Explain why it is or why it isn't. **[20 marks]**

(continued on the other side)

7. A 10 kg block is positioned half-way up a 20 m plane, which is inclined at 30° to the horizontal (e.g., the ground). Gravity is directly pointing downward, and you may neglect friction in this problem.

   (i) What are the forces in the direction normal to and along the plane?

   (ii) When the block is released from the initial position, it will slide down the plane. After how many seconds will the sliding block hit the bottom end of the inclined plane? **[10 marks]**

8. Given a potential, $V(x) = 2x^4 - 4x^2 + 1$,

   (i) Sketch the potential $V(x)$.

   (ii) At which locations ($x$ points) are the local maxima and minima?

   (iii) Indicate the direction of force at each maximum or minimum points on both sides of the points.

   (iv) Where are the local stable and unstable equilibrium points? **[20 marks]**

(end of exam)