

ELECTROMAGNETIC DARK ENERGY

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We introduce a new model for dark energy in the Universe in which a small cosmological constant is generated by ordinary electromagnetic vacuum energy. The corresponding virtual photons exist at all frequencies but switch from a gravitationally active phase at low frequencies to a gravitationally inactive phase at higher frequencies via a Ginzburg–Landau type of phase transition. Only virtual photons in the gravitationally active state contribute to the cosmological constant. A small vacuum energy density, consistent with astronomical observations, is naturally generated in this model. We propose possible laboratory tests for such a scenario based on phase synchronization in superconductors.

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1. Introduction

Current astronomical observations^{1–4} provide compelling evidence that the Universe is presently in a phase of accelerated expansion. This accelerated expansion can be formally associated with a small positive cosmological constant in the Einstein field equations, or more generally with the existence of dark energy. The dark energy density consistent with the astronomical observations is at variance with typical values predicted by quantum field theories. The discrepancy is of the

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order 10^{122} , which is the famous cosmological constant problem.⁵ A large number of theoretical models exist for dark energy in the Universe (see e.g. Refs. 6 and 7 for reviews). It is fair to say that none of these models can be regarded as being entirely convincing, and that further observations and experimental tests^{7–9} are necessary for deciding on the nature of dark energy.

The most recent astronomical observations⁴ seem to favor constant dark energy with an equation of state $w = -1$ as compared to dynamically evolving models. In this paper we introduce a new model for constant dark energy in the Universe which has several advantages relative to previous models. First, the model is conceptually simple, since it associates dark energy with ordinary electromagnetic vacuum energy. In that sense the new physics underlying this model does not require the postulate of new exotic scalar fields such as the quintessence field. Rather, one just deals with particles (ordinary virtual photons) whose existence is experimentally confirmed. Secondly, the model is based on a Ginzburg–Landau type of phase transition for the gravitational activity of virtual photons which for *natural* choices of the parameters generates the correct value of the vacuum energy density in the Universe. In fact, the parameters in our dark energy model have a similar order of magnitude to those that successfully describe the physics of superconductors. Finally, since the phase of the macroscopic wave function that describes the gravitational activity of the virtual photons in our model may synchronize with that of Cooper pairs in superconductors, there is a possibility of testing this electromagnetic-dark-energy model by simple laboratory experiments.

2. Ginzburg–Landau Model of Dark Energy

Recall that quantum field theory formally predicts an infinite vacuum energy density associated with vacuum fluctuations. This is in marked contrast to the observed small positive finite value of dark energy density ρ_{dark} consistent with the astronomical observations. The relation between a given vacuum energy density ρ_{vac} and the cosmological constant Λ in Einstein’s field equations is

$$\Lambda = \frac{8\pi G}{c^4} \rho_{\text{vac}}, \quad (1)$$

where G is the gravitational constant. The small value of Λ consistent with the experimental observations is the well-known cosmological constant problem. Suppression of the cosmological constant using techniques from superconductivity was recently suggested in a paper by Alexander, Mbonye, and Moffat.¹⁰ To construct a simple, physically realistic model of dark energy based on electromagnetic vacuum fluctuations creating a small amount of vacuum energy density, $\rho_{\text{vac}} = \rho_{\text{dark}}$, we assume that virtual photons (or any other bosons) can exist in two different phases: a *gravitationally active* phase where they contribute to the cosmological constant Λ , and a *gravitationally inactive* phase where they do not contribute to Λ .

Let $|\Psi|^2$ be the number density of gravitationally active photons in the frequency interval $[\nu, \nu + d\nu]$. If the dark energy density ρ_{dark} of the Universe is produced by

electromagnetic vacuum fluctuations, i.e. by the zero-point energy term $\frac{1}{2}h\nu$ of virtual photons (or other suitable bosons), then the total dark energy density is obtained by integrating over all frequencies weighted with the number density of gravitationally active photons:

$$\rho_{\text{dark}} = \int_0^\infty \frac{1}{2}h\nu|\Psi|^2 d\nu. \quad (2)$$

The standard choice of

$$|\Psi|^2 = \frac{2}{c^3} \cdot 4\pi\nu^2, \quad (3)$$

in which the factor 2 arises from the two polarization states of photons, makes sense in the low-frequency region but leads to a divergent vacuum energy density for $\nu \rightarrow \infty$. Hence we conclude that $|\Psi|^2$ must exhibit a different type of behavior in the high-frequency region.

In the following we construct a Ginzburg–Landau type theory for $|\Psi|^2$. Our model describes a possible phase transition behavior for the gravitational activity of virtual photons in vacuum, which has certain analogies with the Ginzburg–Landau theory of superconductors (where $|\Psi|^2$ describes the number density of superconducting electrons). It is a model describing dark energy in a fixed reference frame (the laboratory) and is thus ideally suited for experiments that test for possible interactions between dark energy fields and Cooper pairs.⁹

We start from a Ginzburg–Landau free energy density given by

$$F = a|\Psi|^2 + \frac{1}{2}b|\Psi|^4, \quad (4)$$

where a and b are temperature-dependent coefficients. In the following we use the same temperature dependence of the parameters a and b as in the Ginzburg–Landau theory of superconductivity^{11,12}:

$$a(T) = a_0 \frac{1-t^2}{1+t^2}, \quad (5)$$

$$b(T) = b_0 \frac{1}{(1+t^2)^2}. \quad (6)$$

Here t is defined as $t := T/T_c$, T_c denotes a critical temperature, and $a_0 < 0$, $b_0 > 0$ are temperature-independent parameters. Clearly $a > 0$, $b > 0$ for $T > T_c$ and $a < 0$, $b > 0$ for $T < T_c$. The case $T > T_c$ describes a single-well potential, and the case $T < T_c$ a double-well potential.

The equilibrium state Ψ_{eq} is described by a minimum of the free energy density. Evaluating the conditions $F'(\Psi_{\text{eq}}) = 0$ and $F''(\Psi_{\text{eq}}) > 0$, for $T > T_c$ we obtain

$$\Psi_{\text{eq}} = 0, \quad F_{\text{eq}} = 0, \quad (7)$$

whereas for $T < T_c$

$$|\Psi_{\text{eq}}|^2 = -\frac{a}{b}, \quad F_{\text{eq}} = -\frac{1}{2}\frac{a^2}{b}. \quad (8)$$

In the following, we suppress the index $_{\text{eq}}$.

With Eqs. (5) and (6) we may write Eqs. (8) as

$$|\Psi|^2 = -\frac{a_0}{b_0}(1-t^4), \quad (9)$$

$$F = -\frac{a_0^2}{2b_0}(1-t^2)^2. \quad (10)$$

For very small temperatures ($T \ll T_c$) one has $|\Psi|^2 = -a_0/b_0$, which we identify with the low-frequency behavior of photons as given by Eq. (3). Thus

$$-\frac{a_0}{b_0} = \frac{8\pi}{c^3}\nu^2, \quad (11)$$

which leads to

$$|\Psi|^2(T) = \frac{8\pi}{c^3}\nu^2(1-t^4), \quad (12)$$

$$F(T) = \frac{1}{2}a_0\frac{8\pi}{c^3}\nu^2(1-t^2)^2. \quad (13)$$

We also need to formally attribute a temperature T to the virtual photons underlying dark energy. This can be done as follows. Virtual photons have the same energy as ordinary photons in a bath of temperature T if the zero-point energy $\frac{1}{2}h\nu$ satisfies

$$\frac{1}{2}h\nu = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}. \quad (14)$$

This condition is equivalent to

$$h\nu = \Gamma kT, \quad (15)$$

with $\Gamma = \ln 3$. For most of our considerations in the following, the value of the dimensionless constant Γ is irrelevant as the predictions are independent of it.

Using Eq. (15), the critical temperature T_c in the Ginzburg–Landau model now corresponds to a critical frequency $\nu_c = \Gamma kT_c/h$ where the gravitational activity of photons ceases to exist. By putting Eq. (15) into Eqs. (12) and (13) we obtain our final result,

$$|\Psi|^2(\nu) = \frac{8\pi}{c^3}\nu^2 \left(1 - \frac{\nu^4}{\nu_c^4}\right), \quad (16)$$

$$F(\nu) = \frac{1}{2}a_0\frac{8\pi}{c^3}\nu^2 \left(1 - \frac{\nu^2}{\nu_c^2}\right)^2, \quad (17)$$

valid for $\nu < \nu_c$. For $\nu \geq \nu_c$ one has $|\Psi|^2(\nu) = 0$ and $F(\nu) = 0$.

Thus the number density $|\Psi|^2$ of gravitationally active photons in the interval $[\nu, \nu + d\nu]$ is nonzero for $\nu < \nu_c$ only. In this way we obtain a finite dark energy density when integrating over all frequencies:

$$\rho_{\text{dark}} = \int_0^\infty \frac{1}{2}h\nu|\Psi|^2 d\nu \quad (18)$$

$$= \frac{4\pi h}{c^3} \int_0^{\nu_c} \nu^3 \left(1 - \frac{\nu^4}{\nu_c^4}\right) d\nu = \frac{1}{2} \frac{\pi h}{c^3} \nu_c^4 \quad (19)$$

(note the factor $1/2$ as compared to previous work^{13,14}). The currently observed dark energy density in the Universe of about 3.9 GeV/m^3 ¹⁻³ implies that the critical frequency ν_c is given by $\nu_c \approx 2.01 \text{ THz}$.

Note that in our model virtual photons exist (in the usual quantum-field-theoretical sense) for both $\nu < \nu_c$ and $\nu \geq \nu_c$, and hence there is no change either to quantum electrodynamics (QED) or to measurable QED effects such as the Casimir effect at high frequencies. The only thing that changes at ν_c is the *gravitational* behavior of virtual photons. This is a new physics effect at the interface between gravity and electromagnetism,^{15,16} which solely describes the *gravitational* properties of virtual photons.

We may calculate further interesting quantities for this electromagnetic-dark-energy model. The total number density N of gravitationally active photons is

$$N = \int_0^\infty |\Psi|^2(\nu) d\nu \quad (20)$$

$$= \frac{8\pi}{c^3} \int_0^{\nu_c} \nu^2 \left(1 - \frac{\nu^4}{\nu_c^4}\right) d\nu \quad (21)$$

$$= \frac{32}{21} \frac{\pi}{c^3} \nu_c^3. \quad (22)$$

Similarly, the total free energy density is given by

$$F_{\text{total}} = \int_0^\infty F(\nu) d\nu = \frac{32\pi}{105} \frac{1}{c^3} a_0 \nu_c^3. \quad (23)$$

Thus on average the free energy per gravitationally active photon is given by

$$\frac{F_{\text{total}}}{N} = \frac{1}{5} a_0. \quad (24)$$

This result is independent of ν_c and gives a simple physical interpretation to the constant a_0 , which has the dimension of an energy, just as for ordinary superconductors.

Our electromagnetic-dark-energy model depends on two *a priori* unknown parameters — the critical frequency $h\nu_c$ and the constant a_0 . It shares many similarities with the Ginzburg–Landau theory of superconductors, formally replacing the number density of superconducting electrons in the superconductor by the number density of gravitationally active photons in the vacuum. It is instructive to see which values the constants a_0 and $h\nu_c$ take for typical superconductors in solid state physics.

The Bardeen–Cooper–Schrieffer (BCS) theory yields the prediction¹²

$$a_0 = -\alpha kT_c, \quad (25)$$

where

$$\alpha := \frac{6\pi^2}{7\zeta(3)} \frac{kT_c}{\mu}. \quad (26)$$

Here μ denotes the Fermi energy of the material under consideration. For example, in copper $\mu = 7.0$ eV, and the critical temperature of a YBCO (yttrium–barium–copper oxide) high- T_c superconductor is around 90 K. This yields typical values of $h\nu_c \sim 8 \cdot 10^{-3}$ eV and $\alpha \sim 8 \cdot 10^{-3}$.

Remarkably, our dark energy model works well if the free parameters a_0 and $h\nu_c$ have the same order of magnitude as in solid state physics. Many dark energy models suffer from the fact that one needs to input extremely fine-tuned or unnatural parameters. This is not the case for the Ginzburg–Landau-like model described here. Our model is based on analogies with superconductors, and in view of *naturalness* it would seem most plausible that the relevant dark energy parameters have a *similar order of magnitude* as in solid state physics. Moreover, the parameters of our model should be *universal* parameters related to electroweak interactions, since we consider an electromagnetic model of dark energy.

A possible choice is

$$a_0 = -\alpha_{\text{el}} \cdot h\nu_c, \quad (27)$$

where $h\nu_c \sim m_\nu c^2$ is proportional to a typical neutrino mass scale, and $\alpha_{\text{el}} \approx 1/137$ is the fine structure constant. The motivation for (27) is as follows. Since we are considering a model of dark energy based on electromagnetic vacuum energy, the relevant interaction strength should be the electric one described by α_{el} . Moreover, in solid state physics the critical temperature is essentially determined by the energy gap of the superconductor under consideration¹¹ (i.e. the energy obtained when a Cooper pair forms out of two electrons). Something similar could be relevant for the vacuum. We could think that at low temperatures (frequencies) Cooper-pair-like states can form in the vacuum. If this new physics has to do with neutrinos, one would expect that the relevant energy gap would be of the order of typical neutrino mass differences. Solar neutrino measurements provide evidence for a neutrino mass of about $m_\nu c^2 \sim 9 \cdot 10^{-3}$ eV,^{17,18} assuming a mass hierarchy of neutrino flavors. This agrees with the energy scale $h\nu_c$ which we need here to reproduce the correct amount of dark energy density in the Universe.

Another constraint condition in our model of dark energy is that the parameter α should not be too large. Otherwise, one would have in equilibrium a surplus of negative free energy density, which would counterbalance the positive dark energy density. We obtain from Eqs. (23), (19) and (27) the ratio

$$\frac{F_{\text{total}}}{\rho_{\text{dark}}} = -\frac{64}{105} \alpha_{\text{el}}, \quad (28)$$

hence $|F_{\text{total}}| \ll \rho_{\text{dark}}$, as required.

3. Possible Measurable Effects in Superconductors

We now turn to possible measurable effects of our theory. The similarity to the Ginzburg–Landau theory of superconductivity, and in particular the fact that the potential parameters have the same order of magnitude, suggest the possibility

that gravitationally active photons could produce measurable effects in superconducting devices via a possible synchronization of the phases of the corresponding macroscopic wave functions.

Denote the macroscopic wave function of gravitationally active photons by Ψ_G (previously this was denoted as Ψ), and that of superconducting electrons (Cooper pairs) in a superconductor by Ψ_s . So far we only dealt with absolute values of these wave functions, but we now introduce the phases Φ_G and Φ_s by writing

$$\Psi_s = |\Psi_s|e^{i\Phi_s}, \quad (29)$$

$$\Psi_G = |\Psi_G|e^{i\Phi_G}. \quad (30)$$

In superconductors one has $|\Psi_s|^2 = \frac{1}{2}n_s$, where n_s denotes the number density of superconducting electrons. Similarly, in our model $|\Psi_G|^2$ is proportional to the number density of gravitationally active photons. Spatial gradients in the phase Φ_s give rise to electric currents

$$\mathbf{j}_s = \frac{e\hbar}{m}|\Psi_s|^2\nabla\Phi_s = -\frac{ie\hbar}{2m}(\Psi_s^*\nabla\Psi_s - \Psi_s\nabla\Psi_s^*), \quad (31)$$

where e is the electron charge and m the electron mass. Similarly, spatial gradients in the phase Φ_G of gravitationally active photons would generate a current given by

$$\mathbf{j}_G = \frac{\tilde{e}\hbar}{\tilde{m}}|\Psi_G|^2\nabla\Phi_G = -\frac{i\tilde{e}\hbar}{2\tilde{m}}(\Psi_G^*\nabla\Psi_G - \Psi_G\nabla\Psi_G^*). \quad (32)$$

Whereas the strength of the electromagnetic current is proportional to the Bohr magneton $\mu_B = \frac{e\hbar}{2m}$, the strength of the current given by Eq. (32) is proportional to a kind of ‘‘gravitational magneton,’’ $\mu_G := \frac{\tilde{e}\hbar}{2\tilde{m}}$, whose strength is *a priori* unknown. Presumably, μ_G is very small so that this current is normally unobservable in the vacuum.

In superconducting devices, however, the situation may be very different. Here both of the phases Φ_s and Φ_G exist and the corresponding wave functions might interact. The strength of this interaction is *a priori* unknown since Ψ_G represents new physics. If the interaction strength is sufficiently strong, then in equilibrium the phases may synchronize:

$$\Phi_s = \Phi_G. \quad (33)$$

This is plausible because of the similarity of the size of parameters of the corresponding Ginzburg–Landau potentials. If phase synchronization sets in, then fluctuations in Φ_G would produce measurable stochastic electric currents of superconducting electrons given by

$$\mathbf{j}_s = \frac{e\hbar}{m}|\Psi_s|^2\nabla\Phi_G. \quad (34)$$

However, these currents could only exist up to the critical frequency ν_c . For $\nu > \nu_c$ one has $\Psi_G = 0$ and hence $\nabla\Phi_G = 0$. Noise currents that are produced by gravitationally active photons would thus cease to exist at a critical frequency

ν_c given by about 2 THz. Generally, our Ginzburg–Landau model predicts that gravitationally active photons produce a quantum noise power spectrum,

$$S(\nu) = \frac{1}{2}h\nu \left(1 - \frac{\nu^4}{\nu_c^4}\right), \quad (35)$$

for $\nu < \nu_c$ and $S(\nu) = 0$ for $\nu \geq \nu_c$.

In resistively shunted Josephson junctions, quantum noise power spectra induced by stochastically fluctuating phases can be quite precisely measured.¹⁹ For frequencies smaller than 0.5 THz the form of the power spectrum of current fluctuations has been experimentally confirmed¹⁹ to be

$$\hat{S}(\nu) = \frac{4}{R} \left(\frac{1}{2}h\nu + \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \right), \quad (36)$$

a direct consequence of the fluctuation dissipation theorem.^{20,21} Here R denotes the shunt resistor. The first term in Eq. (36) is due to zero-point fluctuations and the second term is due to ordinary thermal noise. If there is full synchronization between the phases Φ_s and Φ_G , then our Ginzburg–Landau model predicts a high-frequency modification of (36) given by

$$\tilde{S}(\nu) = \frac{4}{R} \left[\frac{1}{2}h\nu \left(1 - \frac{\nu^4}{\nu_c^4}\right) + \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \right]. \quad (37)$$

This spectrum agrees with the spectrum (36) up to frequencies of about 1 THz, but it then reaches a maximum at $\nu_{\max} = 5^{-1/4}\nu_c \approx 1.34$ THz and approaches a cutoff at $\nu_c \approx 2.01$ THz (see Fig. 1). New Josephson experiments are currently

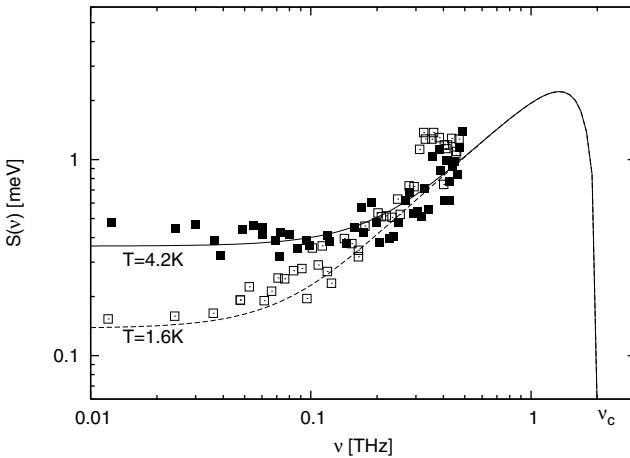


Fig. 1. Power spectrum $S(\nu) := \frac{R}{4}\tilde{S}(\nu)$ as given by Eq. (37) at two different temperatures (solid and dashed lines) and comparison with the data of the experiment of Koch *et al.*¹⁹ Filled squares correspond to measurements at 4.2 K, open squares to measurements at 1.6 K.

being carried out²² that will reach the THz frequency range, thus being able to compare the prediction (37) with the experimental data for frequencies larger than 0.5 THz.

4. Conclusion and Outlook

To conclude, in this paper we have introduced a new model of dark energy where the dark energy of the Universe is identified as ordinary electromagnetic vacuum energy. The new physics of the model consists of a phase transition of virtual photons from a gravitationally active to a gravitationally inactive state. This phase transition is described by a Ginzburg–Landau-like model. The advantage of our model is that it yields the correct amount of dark energy in the Universe for natural choices of the parameters (quite similar to those used in the Ginzburg–Landau theory of superconductors), and that the predictions of our model can be tested by laboratory experiments.

We end with a brief discussion on possible future developments of the model studied here. First, it should be noted that our Ginzburg–Landau approach can be applied to any type of boson and need not be confined to photons. Most generally, we may assume that any particle in the standard model may exist in either a gravitationally active or inactive phase. To describe the phase transition behavior of a given boson, one may consider fundamental fermionic degrees of freedom that can condense into the boson being considered. A more advanced theory would construct the analog of Cooper pairs in solid state physics, with a suitable weak attractive force between the fundamental fermionic degrees of freedom. This would be equivalent to the development of a BCS type of theory that effectively reproduces the Ginzburg–Landau model studied here. In Ref. 10 a similar idea has been developed, with the weak attracting force leading to the condensate being gravity. In other models, for example for neutrino superfluidity, the weak mediating force between massive neutrinos that can lead to superfluid neutrino states is given by Higgs boson exchange.²³ Our model is phenomenological; just as in solid state physics, the Ginzburg–Landau theory arises as a phenomenological model out of the BCS theory. Our model does not rely on a particular microscopic model, but makes universal predictions for measurable spectra as shown in Fig. 1, assuming that the dark energy condensate interacts with ordinary Cooper pairs via a synchronization of phases. If other particle condensates also contribute to the measured spectra, then this would change the degrees of freedom entering into Eq. (3), thus leading to a slightly different critical frequency, ν_c . Future experimental measurements of the critical frequency will thus be a very helpful tool for constraining the class of theoretical models considered.

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