Exercise Sheet 6 — Chaos and Fractals (MTH6107)

due: Thursday, 13 November 2008, 5pm

1. Determine the Liapunov exponent of the map $f(x) = 1 - a|x|$ on the interval $X = [-1, 1]$. a is a parameter that takes on values in $\mathbf{R}^{>0}$. For which values of a does the map exhibit chaotic behaviour? For which values of a does the map have a stable fixed point?

2. Determine the Liapunov exponent of the logistic map $f(x) = 1 - \mu x^2$ in the region $\mu \in (\frac{3}{4}, \frac{5}{4})$, where the map has a stable periodic orbit of period length $L = 2$. Show explicitly that $\lambda < 0$.

3. Consider the map $y = f(x) = 1 - 2\sqrt{|x|}$ on $X = [-1, 1]$.

   a) Draw a graph of $f(x)$. Determine $f'(x)$ and the two branches of the inverse function $f^{-1}(y)$.

   b) Write down the Perron-Frobenius operator for this mapping.

   c) Show that any function of the form $\rho(y) = C \cdot (1 - y)$ (C: a constant) is a fixed point of the Perron-Frobenius operator.

   d) Determine the constant C from the normalization condition $\int_{-1}^{1} \rho(y) dy = 1$.