Complexity of chaotic fields and standard model parameters

Christian Beck
School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, UK

Abstract

In order to understand the parameters of the standard model of electroweak and strong interactions (coupling constants, masses, mixing angles) one needs to embed the standard model into some larger theory that accounts for the observed values. This means some additional sector is needed that fixes and stabilizes the values of the fundamental constants of nature. In these lecture notes we describe in non-technical terms how such a sector can be constructed. Our additional sector is based on rapidly fluctuating scalar fields that, although completely deterministic, evolve in the strongest possible chaotic way and exhibit complex behaviour. These chaotic fields generate potentials for moduli fields, which ultimately fix the fundamental parameters. The chaotic dynamics can be physically interpreted in terms of vacuum fluctuations. These vacuum fluctuations are different from those of QED and QCD but coupled with the same moduli fields as QED and QCD are. The vacuum energy generated by the chaotic fields underlies the currently observed dark energy of the universe. Our theory correctly predicts the numerical values of the electroweak and strong coupling constants using a simple principle, the minimization of vacuum energy. Implementing some additional discrete symmetry assumptions one also obtains predictions for fermion masses, as well as a Higgs mass prediction of 154 GeV.

1 Introduction

String theories predict an enourmous amount of possible vacua after compactification, of the order 10^{120}. In each of these vacua the cosmological constant as well as the fundamental constants of nature can have different values. One is led to the so-called 'landscape' picture [1]. The landscape in a sense represents the set of all possible blueprints of the universe. To select the right vacuum, i.e. the one we observe around us right now, often an anthropic point of view is chosen. But is an anthropic principle really the last word and the ultimate answer to all our questions?

A natural idea to avoid an anthropic selection principle would be that the universe will not be left alone with its choice of 10^{120} vacua but that it gets some help. This help should have the form of an additional sector, a theory as yet not included in the ordinary standard model, neither in ordinary string theories. The additional sector should yield some general principle to select, fix and stabilize the standard model parameters in the way we do observe them. In fact, it should do much more: It should choose the right gauge groups, the correct amount of supersymmetry, the correct number of flavour families, it should create an excess of matter over anti-matter, and so on. The additional sector should fix the most relevant information about the future universe at a very early stage, similar as a DNA string fixes the most important information of a human being already when the first cells are formed.

We are still far away from such a theory. But some numerical observations have recently been made [2, 3, 4] that seem to give us a hint how this additional sector could look like. Our aim here is to explain the relevant concepts in a non-technical way.

In practice standard model parameters can be thought of as being fixed by so-called moduli fields. A varying standard model parameter (e.g. the fine structure constant) can be essentially regarded as (a simple function of) such a moduli field. These moduli fields evolve to minima of their potentials. So if we know the correct moduli potentials describing the world around us, we also know the correct standard model parameters. So what could be a theory to construct these moduli potentials? In principle the potentials should follow from the embedding theory (e.g. M theory + compactification + supersymmetry breaking), but little is known in practice due to the enourmous complexity inherent in the above theories. But as with any unknown theory we can be guided by first trying to find an empirical theory that does the correct thing, i.e. reproduces the observed value of the fine structure constant and other fundamental constants, and then later try to embed it into a greater context. The interesting thing is that such an empirical theory is possible [2, 3, 4]: There is a class of highly nonlinear chaotic dynamical system that seem to reproduce the 'correct' standard model parameters by a simple selection mechanism, the minimization of vacuum energy.

Physically the above chaotic dynamics can be regarded as describing rapidly fluctuating scalar fields associated with vacuum fluctuations. These are vacuum fluctuations different from those of QED or QCD, so what would be the physical embedding? The most natural embedding is to associate the above chaotic vacuum fluctuations with the currently observed dark energy in the universe [4, 5]. Since nobody really knows what dark energy is there is a lot of freedom in the dark energy sector and enough 'space' to embed new things! The chaotic fields, living in the dark energy sector, generate effective potentials for moduli fields — just the same moduli fields that are responsible for the fundamental constants of the standard model of electroweak and strong interactions. The moduli fields then move to the minima of the potentials generated by the chaotic fields, and fix the fundamental constants of nature. The chaotic sector appears to provide a possible answer to the question why we do observe certain numerical values of standard model parameters (such as the fine structure constant or the strong coupling at W-mass scale) in nature, others not. It can be used to avoid anthropic considerations for fundamental constants. Moreover, it generates a small cosmological constant in a rather natural way [4].

These lecture notes are organized as follows: In section 2 we provide some background information on moduli fields, variable fine structure constants and related topics. In section 3 recall the method of stochastic quantization introduced by Parisi and Wu [6, 7]. We then introduce the chaotic fields in section 4, and show how they can generate potentials for moduli fields in section 5. Finally, in section 6 we provide numerical evidence that there are various local minima of the potentials that do reproduce known standard model parameters with high precision. We obtain excellent agreement with measured values seen in collider experiments. We will also make a few predictions for unknown parameters of the standard model, such as the Higgs mass, based on the chaotic field dynamics and some additional symmetry assumptions.

2 Moduli fields and variable fine structure constant

The action of the standard model is well known [8], we don't have to work that out here. The term of the action involving the electromagnetic field tensor $F_{\mu\nu}$ is given by

$$S_F = \int d^4x \sqrt{-g} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right). \tag{1}$$

In models with a variable fine structure constant $\alpha(t)$ [9, 10, 11, 12, 13] the above action is modified as follows:

$$S_F = \int d^4x \sqrt{-g} \left(-\frac{1}{4} B_F(\chi/M^*) F_{\mu\nu} F^{\mu\nu} \right). \tag{2}$$

Here $\chi(t)$ is a homogeneous scalar field, henceforth called the *moduli field* (a related field in string theory is the dilaton field). M^* is a mass of the order of magnitude of the reduced Planck mass. B_F is a function that is in principle determined by the embedding theory (e.g. string theory or M theory). In Bekenstein's model [12],

$$B_F(\chi/M^*) = e^{-2(\chi-\bar{\chi})/M^*}, \tag{3}$$

but other choices are possible as well. Changes in the field $\chi(t)$ imply changes in the fine structure constant $\alpha(t)$. The relation is

$$\frac{\alpha(t)}{\bar{\alpha}} = \frac{1}{B_F(\chi(t)/M^*)} \tag{4}$$

where $B_F(\bar{\chi}/M^*) = 1$. Here $\bar{\alpha}$ is the stationary value of $\alpha(t)$, and $\bar{\chi}$ the stationary value of $\chi(t)$.

The interpretation of the above generalized action (2) is quite obvious: Normally, the electromagnetic field tensor would contribute with the term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ in the action. This relation is now only satisfied in the stationary case. Otherwise the strength of this term is given by a prefactor B_F that depends on the value of the field $\chi(t)$ at a given time t. The fine structure constant $\alpha(t)$ is a simple function of the moduli field $\chi(t)$ according to eq. (4).

For small displacements of the field $\chi(t)$ from its stationary value we may write

$$B_F(\chi(t)/M^*) \approx B_F(\bar{\chi}/M^*) + \frac{\chi(t) - \bar{\chi}}{M^*} B_F'(\bar{\chi}/M^*), \tag{5}$$

where B'_F is the derivative of the function B_F. We thus obtain in leading order from eq. (4)

$$\alpha(t) = \bar{\alpha}\left(1 - B'_F(\bar{\chi}/M^*)\frac{\chi(t)-\bar{\chi}}{M^*}\right). \qquad (6)$$

In Bekenstein's model, $B'_F(\bar{\chi}/M^*) = -2$. Clearly, the fine structure constant approaches its stationary value $\bar{\alpha}$ if the moduli field χ approaches its stationary value $\bar{\chi}$. Using $B'_F(\bar{\chi}/M^*) = -2$ we may also write eq. (6) as

$$\frac{1}{2}M^*\frac{\alpha(t)-\bar{\alpha}}{\bar{\alpha}} = \chi(t) - \bar{\chi} \qquad (7)$$

which shows that the fine structure constant and the moduli field are basically the same.

To introduce a dynamics for the moduli field χ, we need to know its potential $V(\chi)$. In a Robertson-Walker metric the dynamics is then given by

$$\ddot{\chi} + 3H\dot{\chi} + \frac{\partial V}{\partial \chi} = -\xi_m \frac{\rho_m}{M^*}, \qquad (8)$$

where H is the Hubble parameter and ξ_m is the coupling of the field χ to matter (in particular dark matter). ρ_m is the matter density of the universe at a given time. In typical models studied in the literature [9], the coupling ξ_m is very small.

Whereas in general the potential V is unknown, near to the equilibrium point $\bar{\chi}$ we may expand it as

$$V(\chi) = \frac{1}{2}m^2(\chi - \bar{\chi})^2 + const \qquad (9)$$

so that

$$\frac{\partial V}{\partial \chi} = m^2(\chi - \bar{\chi}). \qquad (10)$$

In [9] m is chosen as an extremely small mass parameter, of the order of the current value H_0 of the Hubble parameter:

$$m \sim H_0 \qquad (11)$$

That is to say, one considers an ultralight scalar field χ with a mass of the order 10^{-33} eV. There is motivation for the existence of such ultralight fields from extended supergravity theories [14]. In terms of the fine structure constant $\alpha(t)$, one obtains near the stationary point

$$V(\chi) = \frac{1}{2} m^2 M^{*2} \frac{1}{(B_F'(\bar{\chi}/M^*))^2} \frac{(\alpha(t) - \bar{\alpha})^2}{\bar{\alpha}^2} + const \tag{12}$$

By construction, the energy density $m^2 M^{*2}$ associated with this potential is of the same order of magnitude as the dark energy density of the universe at the present time.

Of course, in a similar way one can introduce further moduli fields corresponding to other standard model coupling constants, masses and mixing parameters as well. The fine structure constant was just one example. In fact, for each relevant standard model parameter there should be a corresponding moduli field. So we need about 20 such fields.

3 Stochastic quantization

Let us now proceed to a 2nd-quantized theory. An elegant method to do 2nd quantization is via the so-called stochastic quantization method. In the Parisi-Wu approach of stochastic quantization one considers a stochastic differential equation evolving in a fictitious time variable s, the drift term being given by the classical field equation [6, 7]. Quantum mechanical expectations correspond to expectations with respect to the generated stochastic processes in the limit $s \to \infty$. The fictitious time s is different from the physical time t, it is just a helpful fifth coordinate to do 2nd quantization. Neglecting spatial gradients the field under consideration is a function of physical time t and fictitious time s. The stochastically quantized equation of motion of a homogeneous scalar field φ in Robertson-Walker metric is

$$\frac{\partial}{\partial s} \varphi = \ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) + L(s,t), \tag{13}$$

where H is the Hubble parameter, V is the potential under consideration and $L(s,t)$ is Gaussian white noise, δ-correlated both in s and t. For e.g. a numerical simulation we may discretize eq. (13) using

$$s = n\tau \tag{14}$$

$$t = i\delta, \tag{15}$$

where n and i are integers and τ is a fictitious time lattice constant, δ is a physical time lattice constant. The continuum limit requires $\tau \to 0$, $\delta \to 0$, but since quantum field theory is expected to break down at the Planck scale, it can make physical sense to keep a small lattice constant of that size as an effective cutoff:

$$\tau \sim \frac{1}{m_{Pl}^2}, \delta \sim \frac{1}{m_{Pl}}. \tag{16}$$

Eq. (13) yields the discrete dynamics

$$\frac{\varphi_{n+1}^i - \varphi_n^i}{\tau} = \frac{1}{\delta^2}(\varphi_n^{i+1} - 2\varphi_n^i + \varphi_n^{i-1}) + 3\frac{H}{\delta}(\varphi_n^i - \varphi_n^{i-1}) + V'(\varphi_n^i) + noise. \tag{17}$$

This can be written as the following recurrence relation for the field φ_n^i

$$\varphi_{n+1}^i = (1-a)\left\{\varphi_n^i + \frac{\tau}{1-a}V'(\varphi_n^i)\right\} + 3\frac{H\tau}{\delta}(\varphi_n^i - \varphi_n^{i-1}) + \frac{a}{2}(\varphi_n^{i+1} + \varphi_n^{i-1}) + \tau \cdot noise, \tag{18}$$

where a dimensionless coupling constant a is introduced as

$$a := \frac{2\tau}{\delta^2}. \tag{19}$$

This coupling a is a free parameter of our stochastically quantized theory. A priori it can take on any value, reminescent of a moduli field.

We may also introduce a dimensionless field variable Φ_n^i by writing $\varphi_n^i = \Phi_n^i p_{max}$, where p_{max} is some (so far) arbitrary energy scale. The above scalar field dynamics is equivalent to a spatially extended dynamical system (a coupled map lattice) of the form

$$\Phi_{n+1}^i = (1-a)T(\Phi_n^i) + \frac{3}{2}H\delta a(\Phi_n^i - \Phi_n^{i-1}) + \frac{a}{2}(\Phi_n^{i+1} + \Phi_n^{i-1}) + \tau \cdot noise, \tag{20}$$

where the local map T is given by

$$T(\Phi) = \Phi + \frac{\tau}{p_{max}(1-a)}V'(p_{max}\Phi). \tag{21}$$

Here the prime means

$$' = \frac{\partial}{\partial\varphi} = \frac{1}{p_{max}}\frac{\partial}{\partial\Phi}. \tag{22}$$

A symmetric dynamics of the form

$$\Phi_{n+1}^i = (1-a)T(\Phi_n^i) + \frac{a}{2}(\Phi_n^{i+1} + \Phi_n^{i-1}) + \tau \cdot noise \tag{23}$$

is obtained if $H\delta << 1$, equivalent to

$$\delta << H^{-1}. \tag{24}$$

This approximation is valid if the universe is much older than the physical time lattice constant δ. In this case the term proportional to H in eq. (20) can be neglected. The local map T depends on the potential V under consideration. Since we restrict ourselves to real scalar fields φ, T is a 1-dimensional map.

Let us summarize the main result of this section: Iterating a recurrence relation of the form (23) is *equivalent* to considering a stochastically quantized scalar field. The relation between the map T and the potential V is

$$V'(\varphi) = \frac{1-a}{\tau}\left\{-\varphi + p_{max}T\left(\frac{\varphi}{p_{max}}\right)\right\}. \tag{25}$$

Integration yields

$$V(\varphi) = \frac{1-a}{\tau}\left\{-\frac{1}{2}\varphi^2 + p_{max}\int d\varphi\, T\left(\frac{\varphi}{p_{max}}\right)\right\} + const, \tag{26}$$

which in terms of the dimensionless field Φ this can be written as

$$V(\varphi) = \frac{1-a}{\tau}p_{max}^2\left\{-\frac{1}{2}\Phi^2 + \int d\Phi T(\Phi)\right\} + const. \tag{27}$$

4 Introducing chaotic fields

We now come to the crucial point, namely what type of dynamics is generated by (23) for various types of scalar field theories. Take the example of an ultralight moduli field χ. For these types of fields $p_{max} \sim M^* \sim m_{Pl}$ is large, whereas $V'(\chi) \sim m^2(\chi - \bar{\chi})$ is extremely small. This means the mapping T in eq. (21) is extremely close to an identity and the field moves very slowly. Effectively this means that our discrete dynamics (23) approximates very well a smooth continuum evolution of the field χ. The field χ smoothly approaches a minimum of the potential and stays there, apart from some small fluctuations induced by the noise term.

Once again, for a moduli field χ the energy scale associated with the field variable is very large, of the order of the Planck mass $M^* \sim m_{Pl}$, whereas the potential V contains an extremely small mass parameter m, of the order of the present Hubble constant H_0. We could ask whether for symmmetry reasons maybe another scalar field φ exists that has just the opposite properties, i.e. the energy scale of that field variable φ is extremely small (of the order H_0) but its potential V contains a mass term of order M^*?

Indeed, such a field has been studied in [4]. Due to the fact that the forcing is now very strong, this field exhibits strongly chaotic behaviour if it is stochastically quantized. This can immediately be seen from eq. (21): With p_{max} being small, the map T is now far away from the identity. Chaotic behaviour is possible. The small noise term in eq. (23) can actually be neglected in the chaotic case.

We may go one step further in our symmetry considerations. The moduli field is as slow and regular as a field can be. What would be the other extreme? What would be a field that is as rapidly fluctuating and irregular as possible? In other words, which scalar field dynamics would create the strongest possible chaotic behaviour?

The above question has been solved in [15, 16] and is well understood from a nonlinear dynamics point of view. One knows that the maps with strongest chaotic properties (being smooth and deterministic at the same time) are those conjugated to a Bernoulli shift [17] (a shift of integer symbols in suitable coordinates). Among those, certain deterministic maps are even more 'random' than others, in the sense of having least higher-order correlations: These are the so-called Tchebyscheff maps T_N of N-th order, defined as

$$T_2(\Phi) = 2\Phi^2 - 1 \qquad (28)$$

$$T_3(\Phi) = 4\Phi^3 - 3\Phi \qquad (29)$$

$$... = ... \qquad (30)$$

$$T_N(\Phi) = \cos(N \arccos \Phi) \qquad (31)$$

They can arise out of the above dynamics (23) for suitable potentials V.

The most important scalar field potential in particle physics is of course a double-well potential. So let us consider the distinguished example of a φ^4-theory generating strongest possible chaotic behaviour. Take the potential

$$V_{-3}(\varphi) = \frac{1-a}{\tau} \left\{ \varphi^2 - \frac{1}{p_{max}^2} \varphi^4 \right\} + const, \qquad (32)$$

or, in terms of the dimensionless field Φ,

$$V_{-3}(\varphi) = \frac{1-a}{\tau} p_{max}^2 (\Phi^2 - \Phi^4) + const. \tag{33}$$

Applying our formalism of the previous section, we end up with the following local map:

$$\Phi_{n+1} = T_{-3}(\Phi_n) = -4\Phi_n^3 + 3\Phi_n \tag{34}$$

T_{-3} is the (negative) third-order Tchebyscheff map. It is conjugated to a Bernoulli shift of 3 symbols, and generates the strongest possible stochastic behaviour possible for a smooth low-dimensional deterministic dynamical system.

Apparently, starting from the potential (32) we obtain by second quantization a field φ that rapidly fluctuates in fictitious time on some finite interval, provided that initially $\varphi_0 \in [-p_{max}, p_{max}]$. The small noise term in eq. (23) can be neglected as compared to the deterministic chaotic fluctuations of the field. We physically interprete these rapid changes of the field φ as representing vacuum fluctuations. Of course these are vacuum fluctuations different from those of QED or QCD. Since the expectation of the vacuum energy associated with the chaotic field is $\langle V_{-3}(\varphi)\rangle \sim p_{max}^2/\tau \sim H_0^2 m_{Pl}^2$, such a chaotic field yields the correct order of magnitude of vacuum energy density in order to account for a small cosmological constant. Hence we assume that the chaotic fields underly dark energy.

The above example of a chaotic φ^4-theory can be generalized. There are various discrete degrees of freedom to introduce a deterministic chaotic field dynamics that is as random as possible. Consider a 1-dimensional lattice, the lattice sites are labelled by integers i. At each lattice site i we have a dimensionless field variable Φ_n^i which evolves in discrete time n. In the uncoupled case, the dynamics is given by

$$\Phi_{n+1}^i = \pm T_N(\Phi_n^i), \tag{35}$$

where $\pm T_N$ is either the positive of the negative Tchebyscheff map. The initial value Φ_0 is chosen on the interval $[-1, 1]$, the iterates Φ_n^i then stay in this interval, but evolve in a deterministic chaotic way. The dynamics is conjugated to a Bernoulli shift of N symbols, which means that in suitable coordinates the iteration process is like shifting symbols in a random symbol sequence. There are further discrete degrees of freedom to do the coupling to the nearest neighbours. Instead of coupling to the variables $\Phi_n^{i\pm1}$ at the neighboured sites as in eq. (23), we could also use the updated variables $\pm T_N(\Phi_n^{i\pm1})$. Since n describes fictitious time, it's really not clear which choice is the correct one. For sure both degrees of freedom exist. All these discrete degrees of freedom can be written in a compact form as follows:

$$\Phi_{n+1}^i = (1-a)T_N(\Phi_n^i) + s\frac{a}{2}(T_N^b(\Phi_n^{i-1}) + T_N^b(\Phi_n^{i+1})), \tag{36}$$

where s is a sign variable taking on the values ±1. The choice $s = +1$ is called 'diffusive coupling', but for symmetry reasons it also makes sense sense to study the choice $s = -1$, which we call 'anti-diffusive coupling'. The integer b distinguishes between the forward and backward coupling form, $b = 1$ corresponds to forward coupling ($T_N^1(\Phi) := T_N(\Phi)$), $b = 0$ to backward coupling ($T_N^0(\Phi) := \Phi$). We consider random initial conditions, periodic boundary conditions and large lattices of size i_{max}.

One can easily check that for odd N the choice of s is irrelevant (since odd Tchebyscheff maps satisfy $T_N(-\Phi) = -T_N(\Phi)$), whereas for even N the sign of s is relevant and a different dynamics arises. Hence, restricting ourselves to $N = 2$ and $N = 3$, in total 6 relevant chaotic scalar field theories arise, characterized by $(N, b, s) = (2, 1, +1), (2, 0, +1), (2, 1, -1), (2, 0, -1)$ and $(N, b) = (3, 1), (3, 0)$. For easier notation, in the following we will label these chaotic field theories as $2A, 2B, 2A^-, 2B^-, 3A, 3B$, respectively.

The important thing to remember from this section is that eq. (36) just describes a degenerated stochastically quantized scalar field with strongest possible chaotic properties. For somebody from the nonlinear dynamics and complexity community, the dynamics (36) represents the most natural thing in the world: It's just a coupled map lattice, a standard example of a spatio-temporal dynamical system [18]. For somebody from the elementary particle physics or string theory community, however, eq. (36) may look somewhat unusual and unfamiliar at first sight. But it's sometimes worth to learn new things!

The dynamics (36) is a dissipative deterministic dynamical system. It exhibits chaotic behaviour and produces information, measured by a positive KS entropy [17]. Dissipative deterministic systems as a model of quantum gravity have also been suggested in [19].

5 Generating potentials for moduli fields

Clearly, if the coupling a in the above chaotic field dynamics (36) is chosen as $a = 0$, then there are no correlations between neighboured lattice sites:

$$\langle \Phi_n^i \Phi_n^{i+1} \rangle = 0 \tag{37}$$

Here the notation $\langle \cdots \rangle$ denotes the expectation value, which can be numerically calculated by iterating the maps for random initial conditions and doing a time average. Note that the index i denotes physical time (in units of the lattice constant δ). If we physically interpret the chaotic fluctuations of the field Φ_n^i as some sort of vacuum fluctuations underlying the dark energy of the universe (see [4] for more details), then the above condition (37) looks physically very reasonable: We want subsequent vacuum fluctuations to be uncorrelated in physical time, because otherwise they wouldn't really describe spontaneous fluctuations.

We may, however, insist on an interacting theory, i.e. $a \neq 0$. Are there still some distinguished couplings $a^* \neq 0$ where we can keep the above condition of a vanishing correlation function in physical time? Of course such a state would again be distinguished as being as random as possible, where this concept is now extended from fictitious time to physical time. There are also a couple of other reasons why one wants a vanishing correlation function, see [3] for more details.

In fact, numerical investigations show that the above states with vanishing correlation exist and distinguish certain coupling constants a^*. These are numerically observed to coincide with known standard model coupling strengths (see next section). In general one can show [2, 3] that the quantity

$$W(a) = \frac{1}{2} \langle \Phi_n^i \Phi_n^{i+1} \rangle \tag{38}$$

can be physically interpreted as the interaction energy of the chaotic field theory under consideration (in suitable units). Hence states of strongest random properties, described by a vanishing correlation function, have vanishing interaction energy.

Besides the interaction energy, there is also another relevant vacuum energy associated with the chaotic fields. This is the self energy $V(a)$, given by

$$V^{(2)}(a) = \langle \Phi \rangle - \frac{2}{3} \langle \Phi^3 \rangle \quad (N = 2), \tag{39}$$

respectively

$$V^{(3)}(a) = \frac{3}{2}\langle\Phi^2\rangle - \langle\Phi^4\rangle \quad (N = 3). \tag{40}$$

For a derivation, see [3]. The self energy describes the vacuum energy associated with the potentials that generate the chaotic dynamics in fictitious time (the additive constant is fixed by some symmetry considerations).

In the following, we will use both the interaction energy and the self energy to generate suitable potentials for moduli fields. Recall that classically the moduli field χ obeys

$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi) = 0, \tag{41}$$

where we neglected possible interactions with dark matter. In the vicinity of the stationary state $\bar{\chi}$, one has $V'(\chi) = m^2(\chi - \bar{\chi})$ and the moduli field is essentially the same as the standard model coupling constant α. According to eq. (7) we have

$$\chi(t) = \frac{M^*}{2\bar{\alpha}}\alpha(t) + \bar{\chi} - \frac{1}{2}M^*. \tag{42}$$

Putting eq. (42) into (41), we obtain an equation for α,

$$\ddot{\alpha} + 3H\dot{\alpha} + V'(\alpha) = 0, \tag{43}$$

where locally the potential is given by

$$V(\alpha) = \frac{1}{2}m^2(\alpha - \bar{\alpha})^2. \tag{44}$$

Eq. (43) is just the equation of a damped harmonic oscillator, provided α is in the vicinity of $\bar{\alpha}$.

The crucial point is now that we assume that *the same* moduli fields are responsible for the coupling constants of the standard model of electroweak and strong interactions and those of the dark sector described by the chaotic fields. This means $a = \alpha$. The chaotic field wants to find a state of strongest possible random properties described by a vanishing correlation function $\langle\Phi_n^i\Phi_n^{i+1}\rangle$. Hence the moduli field χ given by eq. (42) with $\alpha = a$ needs to adjust its value. This can be achieved by choosing in eq. (43) the formal potential[1]

$$V(\alpha) = -m^2\int_0^\alpha d\alpha'\langle\Phi_n^i\Phi_n^{i+1}\rangle. \tag{45}$$

[1] Our sign conventions relate to moduli potentials generated by *positive* Tchebyscheff maps.

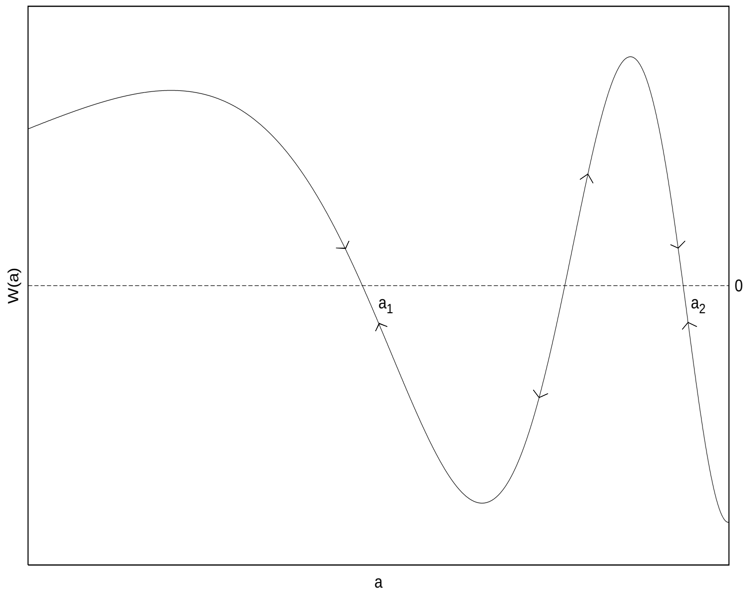

Figure 1: Basic idea underlying this paper. A priori all kinds of standard model couplings $\alpha = a$ (values of moduli fields) are possible. These are then driven into stable zeros of the interaction energy $W(a)$ of the chaotic fields. In the above schematic picture, the stable zeros are denoted by a_1 and a_2.

By construction, local minima of this potential are stable stationary points corresponding to stable (attracting) zeros of the forcing $V'(\alpha) = -m^2 \langle \Phi_n^i \Phi_n^{i+1} \rangle$ (see Fig. 1). If the damping H in eq. (43) is large as compared to m, and if the initial displacement is not too large, then the stationary state $\bar{\alpha}$ is rapidly approached.

For other types of moduli fields, we may choose in eq. (43) the self energy potential $V(\alpha) = m^2 V^{(N)}(\alpha)$ as given in eq. (39) or (40). These types of moduli fields then evolve to local minima of the self energy.

It is well known that standard model interaction strengths actually depend on the relevant energy scale E. We have the running electroweak and strong coupling constants. For example, the fine structure constant $\alpha_{el}(E)$ slightly increases with E, and the strong coupling α_s rapidly decreases with E.

What should we now take for the energy (or temperature) E of the moduli fields near to their stationary state? In other words, if a standard model coupling $\bar{\alpha}$ is fixed as a minimum of the potential $V(\alpha)$, at which energy scale

E is this standard model coupling fixed? *A priori* this is unknown. However, there is extensive numerical evidence [2, 3] that the distinguished couplings $\bar{\alpha} = a^*$ which correspond to minima of the potentials numerically coincide with running standard model coupling constants $\alpha(E)$ with an energy E being given by

$$E = \frac{1}{2}N(m_B + m_{f_1} + m_{\bar{f}_2}). \tag{46}$$

Here N is the index of the Tchebyscheff map considered, and $m_B, m_{f_1}, m_{\bar{f}_2}$ denote the masses of a boson B and a fermion f_1 and anti-fermion $\bar{f}_2$ —not just some exotic bosons and fermions but precisely those that we know from the standard model. One typically observes particle combinations $B, f_1, \bar{f}_2$ that describe possible interaction states in the standard model, for example a decay of the form $B \to f_1 + \bar{f}_2$ or a reaction $f_1 + \bar{f}_2 \to B$. Detailed evidence will be given in the next section. Formula (46) formally reminds us of the energy levels $E_N = \frac{N}{2}\hbar\omega$ of a quantum mechanical harmonic oscillator, with low-energy levels ($N = 2, 3$) given by the masses of the standard model particles.

6 Numerical results

We now present our numerical results (much more details can be found in [2, 3]). In particular, we will show that stable zeros of the correlation functions (states of strongest random properties for chaotic fields, which are stationary states for moduli fields) reproduce known standard model coupling constants and masses with high precision. Our results are obtained by long-term iteration of the chaotic dynamics. One numerically calculates, for a given coupling a, the interaction energy as the time average

$$W(a) = \frac{1}{2}\langle \Phi_n^i \Phi_n^{i+1} \rangle = \frac{1}{2} \lim_{n\to\infty} \lim_{J\to\infty} \frac{1}{MJ} \sum_{n=1}^{M} \sum_{i=1}^{J} \Phi_n^i \Phi_n^{i+1}, \tag{47}$$

where the Φ_n^i evolve according to (36) with random initial conditions. In practice, we used a finite lattice of size $J = 10000$ with periodic boundary conditions, and iteration numbers corresponding to several weeks of CPU time. Everybody is welcome to reproduce and verify the numerical results described below—the recurrence relation (36) can be very easily installed on any computer. What one observes is that the stable zeros of $W(a)$ coincide

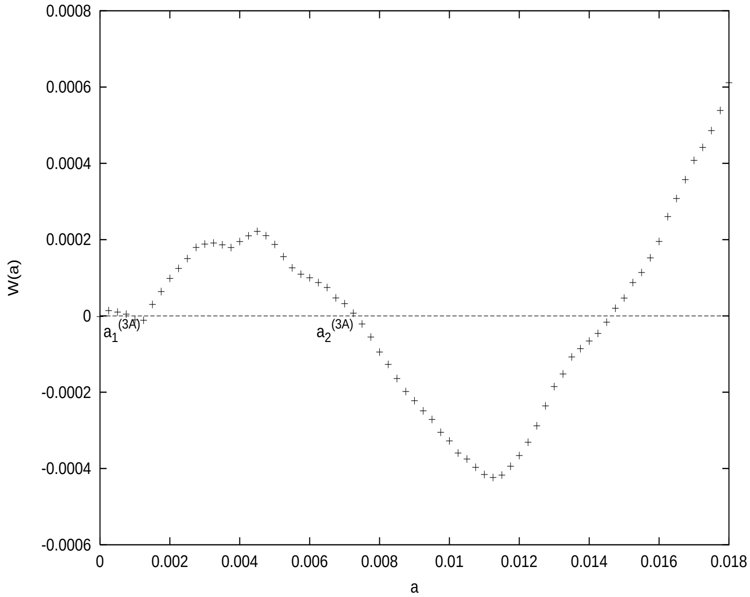

Figure 2: Interaction energy of the 3A dynamics in the small-coupling region.

with known standard model interaction strengths, thus indicating the physical relevance of the theory presented presented in sections 1–5. The chaotic fields appear to determine the moduli potentials in precisely the way we need them to be for a physically realistic vacuum. They can 'help' the universe to find the 'right' vacuum out of an incredibly large number of choices. Anthropic considerations can be avoided in this context.

6.1 The 3A dynamics—electric interaction strengths of electrons and *d*-quarks

Fig. 2 shows the interaction energy $W(a) = \frac{1}{2}\langle \Phi_n^i \Phi_n^{i+1} \rangle$ of the chaotic $3A$ dynamics in the small-coupling region. We observe two stable zeros of the interaction energy in the low-coupling region:

$$
\begin{aligned}
a_1^{(3A)} &= 0.0008164(8) \\
a_2^{(3A)} &= 0.0073038(17)
\end{aligned}
$$

Previously these stable zeros were denoted by a^* or $\bar{\alpha}$. A stable zero satisfies $V'(a^*) = 0$ and $V''(a^*) > 0$, with V given by eq. (45).

Remarkably, the zero $a_2^{(3A)}$ appears to numerically coincide with the fine structure constant $\alpha_{el} \approx 1/137$. To construct a suitable physical interpretation in the sense of eq. (46), let us choose $B = \gamma$, $f_1 = e^-, \bar{f}_2 = e^+$. The relevant energy scale underlying this moduli field is then given by $E = (3/2)(m_\gamma + 2m_e) = 3m_e$, according to eq. (46). Hence our standard model interpretation of the stationary moduli state described by $a_2^{(3A)}$ suggests the numerical identity

$$a_2^{(3A)} = \alpha_{el}(3m_e). \tag{48}$$

For a precise numerical comparison let us estimate the running electromagnetic coupling at this energy scale. We may use the 1st-order QED formula

$$\alpha_{el}(E) = \alpha_{el}(0) \left\{ 1 + \frac{2\alpha_{el}(0)}{\pi} \sum_i f_i \int_0^1 dx\, x(1-x) \log\left(1 + \frac{E^2}{m_i^2} x(1-x)\right)\right\}. \tag{49}$$

The sum is over all charged elementary particles, m_i denotes their (free) masses, and f_i are charge factors given by 1 for e, μ, τ-leptons, $\frac{4}{3}$ for u, c, t-quarks and $\frac{1}{3}$ for d, s, b-quarks. Using this formula, we get $\alpha_{el}(3m_e) = 0.007303$, to be compared with $a_2^{(3A)} = 0.0073038(17)$. There is excellent agreement. Inverting the argument, the above zero of the interaction energy of the chaotic field can be used to predict the numerical value of the fine structure constant from first principles.

Next, we notice that the other zero $a_1^{(3A)}$ has approximately the value $\frac{1}{9}\alpha_{el}$. This could mean that the chaotic 3A field also has a mode that provides evidence for electrically interacting d-quarks. Our interpretation is

$$a_1^{(3A)} = \alpha_{el}^d(3m_d) = \frac{1}{9}\alpha_{el}(3m_d), \tag{50}$$

where $\alpha_{el}^d = \frac{1}{9}\alpha_{el}$ denotes the electromagnetic interaction strength of d-quarks. In the harmonic oscillator interpretation (46), we may choose $B = \gamma$, $f_1 = d, f_2 = \bar{d}$. Formula (49), as an estimate, yields for $m_d \approx 9$ MeV the value $\alpha_{el}(3m_d) = 0.007349$, which coincides very well with $9a_1^{(3A)} = 0.007348(7)$. The value $9a_1^{(3A)}$ actually translates to the energy scale $E_d = (26.0 \pm 6.4)$ MeV. This yields $m_d = \frac{1}{3}E_d = (8.7 \pm 2.1)$ MeV, which coincides with estimates of the $\overline{MS}$ current quark mass of the d quark at the proton mass renormalization scale.

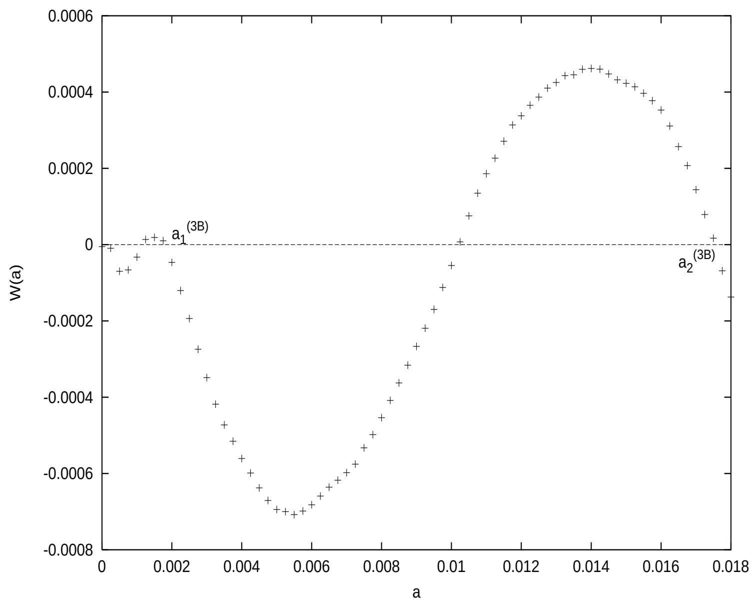

Figure 3: Interaction energy of the 3B dynamics in the low-coupling region.

6.2 The 3B dynamics —weak interaction strengths of neutrinos and u-quarks

The interaction energy $W(a)$ of the 3B field is plotted in Fig. 3. In the low-coupling region $a \in [0, 0.018]$ we observe the following stable zeros of $W(a)$:

$$\begin{aligned} a_1^{(3B)} &= 0.0018012(4) \\ a_2^{(3B)} &= 0.017550(1) \end{aligned}$$

If our approach is consistent, we should be able to find an interpretation of $a_1^{(3B)}$ and $a_2^{(3B)}$ in terms of moduli fields fixing the standard model coupling strengths of u-quarks and neutrinos.

Let us start with $a_2^{(3B)}$. For left-handed neutrinos, the weak coupling due to the exchange of Z^0-bosons is given by

$$\alpha_{weak}^{\nu_L} = \alpha_{el} \frac{1}{4 \sin^2 \theta_W \cos^2 \theta_W}. \tag{51}$$

Here θ_W is the weak mixing angle. In the following we will treat $\sin^2 \theta_W$ as an effective constant, and regard α_{el} as the running electromagnetic coupling. Other renormalization schemes are also possible, but yield only minor numerical differences. Experimentally, the effective weak mixing angle is measured as $\sin^2\theta_W = \bar{s}_l^2 \approx 0.2315$ [20]. Assuming that in addition to the left-handed neutrino interacting weakly there is an electron interacting electrically, the two interaction processes can add up independently if the electron is right-handed, since right-handed electrons cannot interact with left-handed neutrinos. Hence a possible standard model interpretation of the zero $a_2^{(3B)}$ would be

$$a_2^{(3B)} = \alpha_{el}(3m_e) + \alpha_{weak}^{\nu_L}(3m_{\nu_e}) = a_2^{(3A)} + \alpha_{el}(3m_{\nu_e})\frac{1}{4\sin^2\theta_W\cos^2\theta_W} \quad (52)$$

In the harmonic oscillator interpretation of eq. (46), we choose B massless, $f_1 = \nu_L$, $\bar{f}_2 = \bar{\nu_L}$ in addition to the process already described by $a_2^{(3A)}$. Putting in the experimentally measured value of $\sin^2\theta_W = 0.2315$, we obtain for the right-hand side of eq. (52) the value 0.01756, which coincides to 4 digits with the observed stationary value of the moduli field $a_2^{(3B)} = 0.01755$.

Next, let us interpret $a_1^{(3B)}$. In analogy to the joint appearance of ν and e, we should also expect to find evidence for a weakly interacting u-quark, together with a d-quark interacting electrically. Clearly, the u-quark could also interact electrically, but for symmetry reasons we expect the pair (u,d) to interact in a similar way as (ν, e). A right-handed u-quark interacts weakly with the coupling

$$\alpha_{weak}^{u_R} = \frac{4}{9}\alpha_{el}\frac{\sin^2\theta_W}{\cos^2\theta_W}. \quad (53)$$

Adding up the electrical interaction strength of a d-quark, a natural interpretation, quite similar to that of the zero $a_2^{(3B)}$, is

$$a_1^{(3B)} = \alpha_{el}^d(3m_d) + \alpha_{weak}^{u_R}(3m_u) = a_1^{(3A)} + \frac{4}{9}\alpha_{el}(3m_u)\frac{\sin^2\theta_W}{\cos^2\theta_W} \quad (54)$$

The harmonic oscillator interpretation of this moduli state is B massless, $f_1 = u_R$, $f_2 = \bar{u_R}$ in addition to the process underlying $a_1^{(3A)}$. Numerically, taking $\sin^2\theta_W = 0.2315$ and evaluating the running α_{el} using $m_u \approx 5$ MeV, we obtain for the right-hand side of eq. (54) 0.001800, which should be compared with $a_2^{(3B)} = 0.001801$. Again we have perfect agreement within the first 4 digits. It is remarkable that the same universal effective value $\sin^2\theta_W = 0.2315$ can be used consistently for both leptons (couplings $a_2^{(3A)}$,

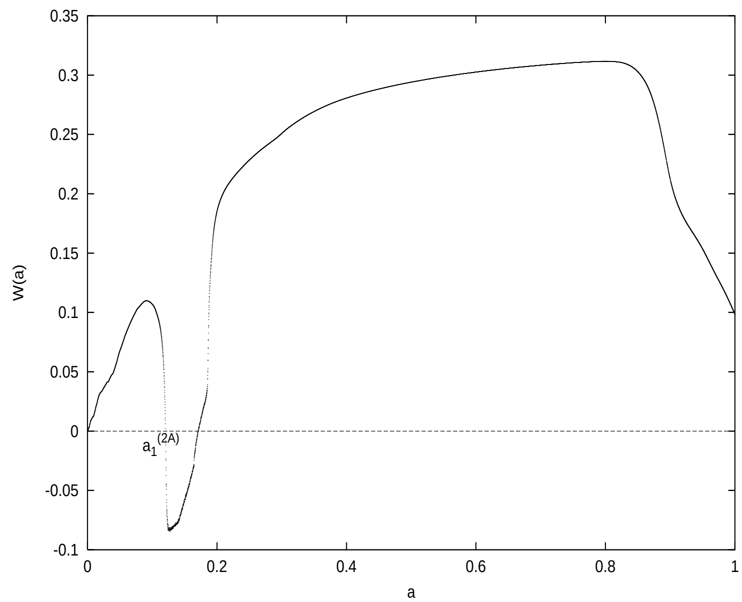

Figure 4: Interaction energy of the 2A dynamics.

$a_2^{(3B)}$) and quarks (couplings $a_1^{(3A)}$, $a_1^{(3B)}$). Inverting the formulas, the observed stationary values of the moduli fields can be used to predict that the weak mixing angle is $\bar{s}_l^2 \approx 0.2315$.

Note that generally the backward coupling form of the $N = 3$ chaotic fields seems to describe a spinless state (formed by e_R and ν_L, respectively d_L and u_R), whereas the forward coupling form just describes one particle species with non-zero spin (e or d). A similar statement will turn out to hold for the $N = 2$ theories, replacing fermions by bosons.

6.3 The 2A dynamics —strong interaction strength at the W-mass scale

If electroweak coupling strengths are fixed by suitable moduli potentials generated by chaotic fields, then something similar should also be the case for the strong coupling strength α_s. Let us now look at chaotic fields with $N = 2$. Fig. 4 shows the interaction energy $W(a)$ of the 2A dynamics. Only one stable zero is observed:

$$a_1^{(2A)} = 0.120093(3) \tag{55}$$

We notice that it numerically seems to coincide with the strong coupling constant α_s at the W- or Z mass scale, which is experimentally measured as $\alpha_s(m_Z) \approx 0.118$.

For symmetry reasons, it seems plausible that if the $N = 3$ dynamics fixes the electroweak couplings at the smallest fermionic mass scales, then the $N = 2$ dynamics could fix the strong couplings at the smallest bosonic mass scales. The lightest massive gauge boson is indeed the $W^\pm$. Hence our physical interpretation associated with $a_1^{(2A)}$ would be $B = W^\pm$, $f_1 = u$, $\bar{f}_2 = \bar{d}$ (respectively $f_1 = d, \bar{f}_2 = \bar{u}$), and since $N = 2$ formula (46) implies

$$a_1^{(2A)} = \alpha_s(m_W + m_u + m_d) \approx \alpha_s(m_W). \tag{56}$$

Since the W-mass is known with high precision, eq. (56) and (55) yield quite a precise prediction for the strong coupling α_s. We can evolve it with high precision to arbitrary energy scales, using the well known perturbative formulas from QCD, obtaining

$$\alpha_s(m_{Z^0}) = 0.117804(12). \tag{57}$$

This prediction of α_s from the zero of the chaotic 2A dynamics is clearly consistent with the experimentally measured value 0.118 and in fact much more precise than current experiments can verify.

6.4 The 2B dynamics—the lightest scalar glueball

The interaction energy of the 2B dynamics is shown in Fig. 5. $W(a)$ has only one non-trivial zero

$$a_1^{(2B)} = 0.3145(1). \tag{58}$$

It is a stable zero, so it should describe an observable stable stationary state of a moduli field. One possibility is to interpret this as a strong coupling at the lightest glueball mass scale. The lightest scalar glueball has spin $J^{PC} = 0^{++}$ and is denoted by gg^{0++} in the following. In our oscillator interpretation we take $B = gg^{0++}$, $f_1 = u$, $\bar{f}_2 = \bar{u}$, thus

$$a_1^{(2B)} = \alpha_s(m_{gg^{0++}} + 2m_u) \approx \alpha_s(m_{gg^{0++}}). \tag{59}$$

The 2B dynamics then describes two bosons at the same time (two gluons forming a glueball), similar to the 3B dynamics, which described two fermions at the same time (left and right handed). In both cases a spin 0 state is

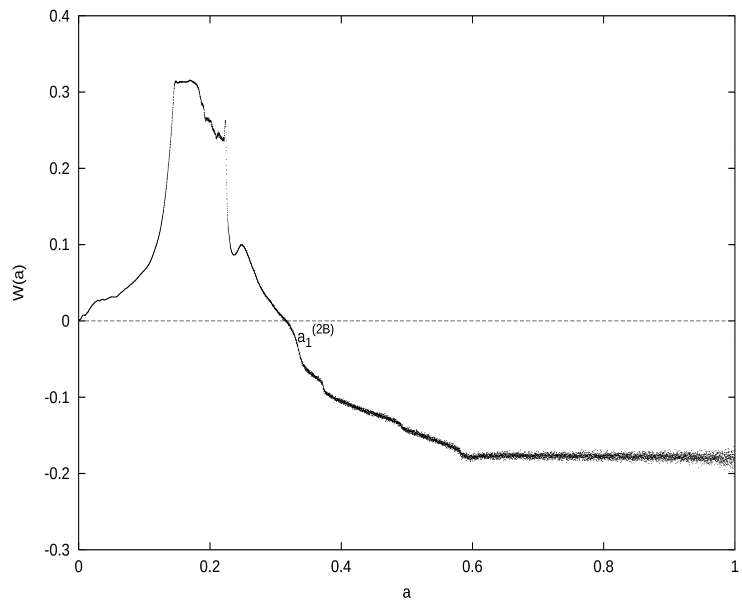

Figure 5: Interaction energy of the 2B dynamics.

formed in total. In lattice gauge calculations including dynamical fermions the smallest scalar glueball mass is estimated as $m_{gg^{0++}} = (1.74 \pm 0.07)$ GeV [21] and at this energy the running strong coupling constant is experimentally measured to be $\alpha_s \approx 0.32$. This clearly is consistent with the observed value of $a_1^{(2B)}$.

6.5 The 2A$^-$ and 2B$^-$ dynamics — towards a Higgs mass prediction

Two chaotic field theories are still remaining, namely those with $N = 2$ and antidiffusive coupling. The interaction energies $W(a)$ of the 2A$^-$ and 2B$^-$ chaotic fields are shown in Fig. 6 and 7. Let us now try to find a suitable physical interpretation for the observed smallest stable zeros $a_1^{(2A^-)} = 0.1758(1)$ and $a_1^{(2B^-)} = 0.095370(1)$. Again let us be guided by discrete symmetry considerations when attributing the various stationary moduli states to standard model particles. We saw that the smallest stable zero of the 2A dynamics describes a boson with non-zero spin (the lightest massive gauge boson $W^\pm$) and the smallest stable zero of the 2B dynamics a boson without spin (the

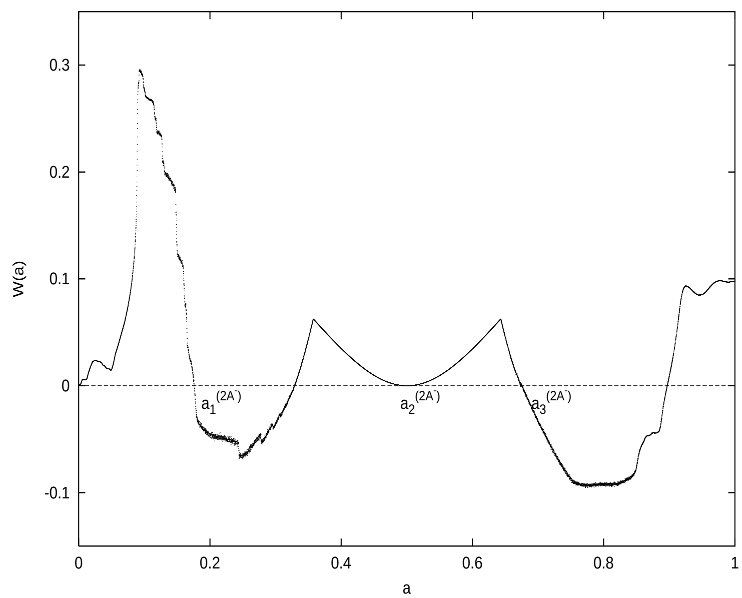

Figure 6: Interaction energy of the $2A^-$ dynamics.

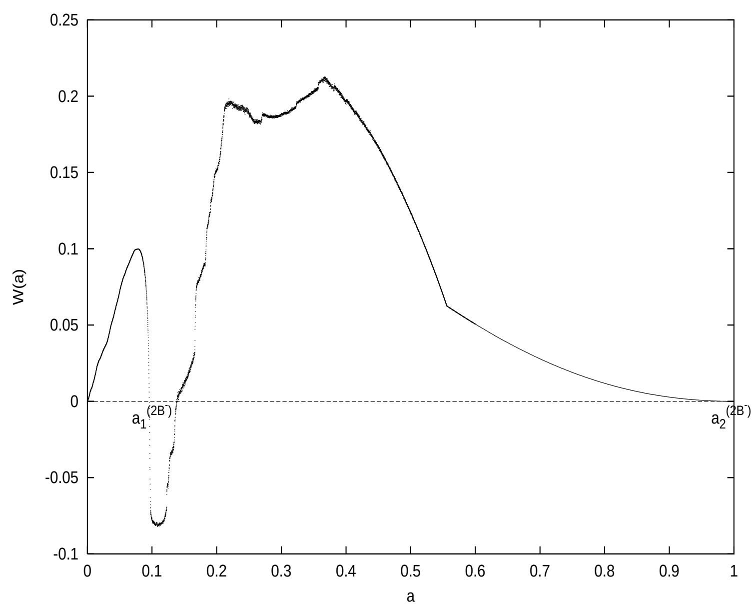

Figure 7: Interaction energy of the $2B^-$ dynamics.

lightest scalar glueball). Thus it seems reasonable to assume that the smallest stable zero of the 2A$^-$ gives us information on yet another bosonic particle with non-zero spin, possibly the lightest glueball with spin $J^{PC} = 2^{++}$, and the smallest zero of the 2B$^-$ dynamics on yet another bosonic particle with spin 0, possibly the lightest Higgs boson.

Let us start with $a_1^{(2A^-)}$. Our physical interpretation of this stationary moduli field state in the sense of eq. (46) is $B = gg^{2++}$, $f_1 = q$, $\bar{f}_2 = \bar{q}$, where $q, \bar{q}$ are suitable quarks. From the strong coupling interpretation

$$a_1^{(2A^-)} = \alpha_s(E_1) \tag{60}$$

the energy $E_1 = m_{gg^{2++}} + 2m_q$ can again be determined from the usual QCD formula, evolving our previously determined $\alpha_s(m_W)$ to lower energies. One obtains $E_1 \approx 10.45$ GeV. In lattice gauge calculations the mass of the lightest 2^{++} glueball is estimated as being roughly 2 GeV. We thus get the correct order of magnitude of the 2^{++} glueball mass if we assume that the quarks in eq. (46) are bottom quarks. With this interpretation a glueball mass $m_{gg2^{++}} = (10.45 - 2 \cdot 4.23)$ GeV =2.0 GeV is predicted, using $m_b = 4.23$ GeV.

Next, let us consider $a_1^{(2B^-)}$. Having already obtained information on the W boson in section 6.3, it is likely that there are also moduli fields that encode the Higgs boson. Our physical interpretation is $B = H$, $f_1 = q$, $\bar{f}_2 = \bar{q}$, where H is the lightest Higgs boson and $q, \bar{q}$ are suitable quarks. The strong coupling interpretation

$$a_1^{(2B^-)} = \alpha_s(E_2) \tag{61}$$

yields $E_2 = 483.4(3)$ GeV $= m_H + 2m_q$. However, experimental and theoretical arguments imply that the Higgs mass should be in the region 100...200 GeV. Hence we only obtain a consistent value for the Higgs mass if we assume that the quarks involved are t quarks. This is similar to the zero $a_1^{(2A^-)}$, where the quarks involved were also heavy quarks. Generally, chaotic fields with antidiffusive couplings seem to encode information on heavy quarks rather than light ones.

From the self energy of the 2B dynamics, one can obtain quite a precise prediction of the free top mass, namely $m_t = 164.5(2)$ GeV, corresponding to a top pole mass of 174.4(3) GeV (see [2, 3] for more details). With this value the zero $a_1^{(2B^-)}$ yields a Higgs mass prediction of

$$m_H = E_2 - 2m_t = 154.4(5) GeV. \tag{62}$$

This is a very precise prediction, the statistical error is very small. But of course the main source of uncertainty is a theoretical uncertainty, namely whether our harmonic oscillator interpretation (in the sense of eq. (46)) of the zero $a_1^{(2B^-)}$ in terms of a Higgs boson and two top quarks is correct. For example, assuming that a supersymmetric extension of the standard model is correct, then the zero could also describe other particle states whose masses add up to 483.4 GeV.

7 New physics?

A few stable zeros of interaction energies remain that cannot be interpreted in the standard model context. The $3A$ dynamics has yet another stable zero $a_3^{(3A)} = 0.07318$. Interpreted as a running strong coupling, this would correspond to an energy $E = \frac{3}{2}(m_B + m_{f_1} + m_{\bar{f}_2}) \approx 7.85$ TeV. One could speculate that this might describe a (rather large) supersymmetric particle scale. There are a few further stable zeros in the large coupling region: $a = 0.9141$ and $a = 1$ $(3A)$, $a = 0.3496$ $(3B)$, $a = 0.675$ $(2A^-)$ and $a = 1$ $(2B^-)$. Possibly these stationary moduli states could describe gravitational couplings at the Planck scale.

So far we only talked about the interaction energy of chaotic fields. But one can also look at the self energy as given in eq. (39) or (40). Again a large number of local minima are observed that can be associated with known standard model interaction strengths, the energy E being again given by eq. (46). We don't have the space to describe all the details here, but refer to [2, 3]. From local minima of the self energy one can get some rather precise predictions of fermion masses, in particular for the heavy fermions t, b, c, τ. The 'landscape' generated by the self energy has also minima that can be identified with Yukawa couplings and gravitational couplings.

The lighter fermions are much more difficult to deal with in this context. Since for very small couplings the self energy exhibits scaling behaviour with log-periodic oscillations [22], one is only able to give predictions of light fermion masses modulo 2, and there are also some other theoretical ambiguities on how to associate the various minima with the light particles. For an early attempt to predict neutrino masses, see [2]. Clearly more work is needed to eliminate the ambiguities for light fermions.

Proceeding to higher energies (by investigating the chaotic dynamics for coupling constants a that coincide with running standard model coupling constants couplings $\alpha_1, \alpha_2, \alpha_3$ at large energies), and using again the self energy as the relevant observable, one can also look for local minima that potentially could describe supersymmetric particles with masses in the TeV region. No such minima are found [3]. Generally the evidence for supersymmetric particles in the TeV region from our chaotic fields is pretty meagre, to say the least. One would expect to see lots of local minima in the relevant coupling region if the conventional ideas on supersymmetry breaking at the TeV scale are correct. The self energies of the chaotic fields do not provide any evidence for such particles: There are simply no minima in the relevant coupling region, the only exception being perhaps the zero $a_3^{(3A)}$ of the 3A interaction energy. However, minima *are* found at the energy scale 10^{16} GeV and at the Planck scale. See [3] for more details. If future accelerator experiments do not find any evidence for supersymmetric particles, a theoretical reason could be that they do not fit into the 'landscape' generated by the chaotic fields.

Talking about new physics, new physics is of course represented by the existence of the chaotic fields themselves. For small couplings a, their equation of state is close to $w = -1$ [4], hence they can account for dark energy in the universe. But for large couplings they can have an equation of state close to $w \approx 0$ [4]. Thus, in principle at least, chaotic fields could account for both, dark energy *and* dark matter in the universe.

8 Conclusion

We have introduced chaotic scalar fields of as a model of vacuum fluctuations in the dark energy sector. These chaotic fields were used to generate potentials for moduli fields. We numerically observe that minima of the potentials lead to realistic standard model coupling strengths. The values of the fine structure constant, of the weak mixing angle, and of the strong coupling at the W mass scale are obtained with high precision and correspond to stable stationary values of moduli fields. Based on additional discrete symmetry assumptions, a value of the Higgs mass of 154 GeV is predicted from the chaotic field dynamics. This prediction can be experimentally tested in the near future.

The theory described in this paper may be regarded as a 'tip of an iceberg'. It still needs to be embedded into a greater context. In particular, its possible relation to supersymmetry breaking mechanisms has to be clarified. However, one statement can be made without any doubt: The nonlinear dynamics given by eq. (36) appears to distinguish certain numerical values of coupling constants a that do coincide with known standard model coupling constants with very high precision. A random coincidence can really be excluded. In this way the chaotic fields can help to select the 'correct' vacuum out of an enormous number of possibilities, shaping the world around us in precisely the way we know it.

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