# Laboratory tests on dark energy

## Christian Beck

School of Mathematical Sciences, Queen Mary, University of London, Mile End Road, London E1 4NS, UK\*

The physical nature of the currently observed dark energy in the universe is completely unclear, and many different theoretical models co-exist. Nevertheless, if dark energy is produced by vacuum fluctuations then there is a chance to probe some of its properties by simple laboratory tests based on Josephson junctions. These electronic devices can be used to perform 'vacuum fluctuation spectroscopy', by directly measuring a noise spectrum induced by vacuum fluctuations. One would expect to see a cutoff near 1.7 THz in the measured power spectrum, provided the new physics underlying dark energy couples to electric charge. The effect exploited by the Josephson junction is a subtile nonlinear mixing effect and has nothing to do with the Casimir effect or other effects based on van der Waals forces. A Josephson experiment of the suggested type will now be built, and we should know the result within the next 3 years.

## I. INTRODUCTION—WHAT IS DARK ENERGY?

It would be nice to start this paper with a clear definition of what dark energy is and what it is good for. Unfortunately, the answer to this question is completely unclear at the moment. What is clear is that various astronomical observations [1, 2] (supernovae, CMB fluctuations, large-scale structure) provide rather convincing evidence that around 73% of the energy contents of the universe is a rather homogeneous form of energy, so-called 'dark energy'. It behaves similar to a cosmological constant and currently causes the universe to accelerate its expansion. Dark energy may just be vacuum energy (with an equation of state  $w = p/\rho = -1$ , where p denotes the pressure and  $\rho$  the energy density). In that case its energy density  $\rho$  is constant and does not change with the expansion of the universe. Or, w may be just close to -1, in which case the dark energy density evolves dynamically and changes with the expansion of the universe. The remaining contents of the current universe is about 23% dark matter and 4% ordinary matter. With 96% of the universe being unknown stuff, there is enough room (and, indeed, the need) for new theories. It seems that in order to construct a convincing theory of dark energy that explains why it is there and what role it plays in the universe one has to be open-minded to new physics.

A large number of different theoretical models exist for dark energy, but an entirely convincing theoretical breakthrough has not yet been achieved. Popular models are based on quintessence fields, phantom fields, quintom fields, Born-Infeld quantum condensates, the Chaplygin gas, fields with nonstandard kinetic terms, to name just a few (see e.g. [3, 4, 5, 6] for reviews). All of these approaches contain 'new physics' in one way or another, though at different levels. However, it is clear that the number of possible dark energy models that are based on new physics is infinite, and in that sense many other models can be considered as well. Only experiments will ultimately be able to confirm or refute the various theoretical constructs.

## II. DARK ENERGY FROM VACUUM FLUCTUATIONS

A priori the simplest explanation for dark energy would be to associate it with vacuum fluctuations that are allowed due to the uncertainty relation. From quantum field theory it is well known that virtual momentum fluctuations of a particle of mass m and spin j formally produce vacuum energy given by

$$\rho_{vac} = \frac{1}{2} (-1)^{2j} (2j+1) \int_{-\infty}^{+\infty} \frac{d^3k}{(2\pi)^3} \sqrt{\mathbf{k}^2 + m^2}$$
 (1)

in units where  $\hbar = c = 1$ . Here **k** represents the momentum and the energy is given by  $E = \sqrt{\mathbf{k}^2 + m^2}$ . Unfortunately, the integral is divergent: It formally yields infinite vacuum energy density. Hence one has to find excuses why this type of vacuum energy is not observable, or why only a very small fraction of it survives.

A typical type of argument is that vacuum energy as given by eq. (1) is not gravitationally active, as long as the theory is not coupled to gravity. This is more a belief rather than a proved statement, and the statement looks a bit

<sup>\*</sup>Electronic address: c.beck@gmul.ac.uk

fragile: Most particles in the standard model of electroweak and strong interactions do have mass, so they know what gravity is and hence it is not clear why their vacuum energy should not be creating a gravitational effect if their mass does. Another argument is that the vacuum energy might be cancelled by some unknown symmetry, for example some type of supersymmetry. For supersymmetric models, fermions with the same mass as bosons generate vacuum energies of opposite sign, so all vacuum energies add up to zero. Unfortunately, we know that we currently live in a universe where supersymmetry is broken: Nobody has ever observed bosonic electrons, having the same mass as ordinary electrons. So this idea does not work either. We see that in both cases the problem why we can't honestly get rid of the vacuum energy comes from the masses of the particles, and there is no theory of masses so far. In fact the Higgs mechanism, which is supposed to create the particle masses in the standard model of electroweak and strong interactions, creates an additional (unacceptable) amount of vacuum energy, due to symmetry breaking. So the only thing that we can say for sure is that the problem of masses and vacuum energy in the current universe is not yet fully understood.

The next, more pragmatic, step is then to introduce an upper cutoff for the momentum  $|\mathbf{k}|$  in the integral (1). This leads to finite vacuum energy. But typically, one would expect this cutoff to be given by the Planck mass  $m_{Pl}$ , since we expect quantum field theory to be replaced by a more sophisticated theory at this scale. Unfortunately, this gives a vacuum energy density of the order  $m_{Pl}^4$ —too large by a factor  $10^{120}$  as compared to current measurements of dark energy density. This is the famous cosmological constant problem. The presently observed dark energy density is more something of the order  $m_{\nu}^4$ , where  $m_{\nu}$  is a typical neutrino mass scale.

To explain the currently observed small dark energy density in the universe, it seems one has to be open-minded to new physics. This new physics can enter at various points. If vacuum fluctuations (of whatever type) create dark energy, we basically have two possibilities: Either, the dark energy is created by vacuum fluctuations of ordinary particles of the standard model (e.g. photons). The corresponding vacuum energy is given by the integral (1), and the new physics should then explain why there is a cancellation of the vacuum energy if the momentum  $|\mathbf{k}|$  exceeds something of the order  $m_{\nu}$ . Or, there may be new types of vacuum fluctuations created by a new type of dynamics underlying the cosmological constant which intrinsically has a cutoff at around  $m_{\nu}$ . A priori, there is no reason why this new dynamics of vacuum fluctuations should not couple to electric charge. In fact, ultimately there is the need to unify the quantum field theory underlying the standard model with gravity, so the missing piece in this jigsaw should couple to both. If the new dynamics of vacuum fluctuations (or the new cancellation process of vacuum energy) couples to electric charge, then there is a chance to see effects of this in laboratory experiments on the earth [7], as we shall work out in the following.

#### III. VACUUM FLUCTUATION SPECTROSCOPY WITH JOSEPHSON JUNCTIONS

It is well known that vacuum fluctuations produce measurable noise in dissipative systems (see, e.g., [8] and references therein). This noise has been experimentally verified in many experiments. What is the deeper reason for the occurence of quantum noise, e.g., in resistors? The basic principle is quite easy to understand. Consider a dissipative system and two canonically conjugated variables, say x (position) and p (momentum). If there are no external forces, then in the long-term run the system will classically reach the stable fixed point p = 0, since all kinetic energy is dissipated. But quantum mechanically, a state with p = 0 would contradict the uncertainty principle  $\Delta x \Delta p = O(\hbar)$ . Hence there must be noise in the resistor that keeps the momentum going. This noise acts as a fluctuating driving force and makes sure that the momentum does not take on a fixed value, so that the uncertainty principle is satisfied at all times.

Quantum noise is a dominant effect if the temperature is small. It can be directly measured in experiments. Maybe one of the most impressive experiments in this direction is the one done by Koch et al. in the early eighties, based on resitively shunted Josephson junctions [9, 10]. A Josephson junction consists of two superconductors with an insulator sandwiched inbetween. The behaviour of such a device can be modeled by the following stochastic differential equation

$$\frac{\hbar C}{2e}\ddot{\delta} + \frac{\hbar}{2eR}\dot{\delta} + I_0 \sin \delta = I + I_N. \tag{2}$$

Here  $\delta$  is the phase difference across the junction, R is the shunt resistor, C the capacitance of the junction, I is the mean current,  $I_0$  the noise-free critical current, and  $I_N$  is the noise current. One can think of eq. (2) as formally describing a particle that moves in a tilted periodic potential. The noise current  $I_N$  produces some perturbations to the angle of the tilt. There are two main sources of noise: Thermal fluctuations and quantum fluctuations.

Koch et al. [10] measured the noise spectrum at low temperatures using four different Josephson devices. They

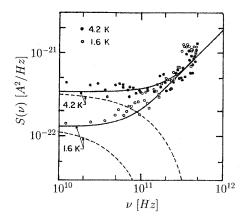


FIG. 1: Spectral density of current noise as measured in Koch et al.'s experiment [10] for two different temperatures. The solid line takes into account the zero-point term and is given by eq. (3), whereas the dashed line is given by a purely thermal noise spectrum,  $(4h\nu/R)(exp(h\nu/kT)-1)^{-1}$ .

experimentally verified the following form of the spectrum up to frequencies of  $6 \cdot 10^{11} Hz$ :

$$S(\nu) = \frac{2h\nu}{R} \coth\left(\frac{h\nu}{2kT}\right)$$

$$= \frac{4h\nu}{R} \left(\frac{1}{2} + \frac{1}{\exp(h\nu/kT) - 1}\right). \tag{3}$$

The first term in Equation (3) grows linear with the frequency, as expected for the ground state of a quantum mechanical oscillator. This term represents quantum noise and is induced by zero-point fluctuations. The second term is ordinary Bose-Einstein statistics and corresponds to thermal noise.

What is remarkable with the experiment of Koch et al. is the fact that the quantum fluctuations produce a directly measurable spectrum—many theoretical physicists thought (and some still think) that this is not possible. The experimentally measured data of Koch et al. are shown in Fig. 1. In fact, noise induced by quantum fluctuations plays an important role in any resistor if the temperature is small enough. The Josephson junction serves as a useful technical tool in this context: Due to a nonlinear mixing effect in the junction, one has the experimental possibility to obtain measurements of very high frequency noise. For recent theoretical work on the quantum noise theory of Josephson junctions, see [8, 11].

The linear term in the spectrum is induced by zero-point fluctuations and thus a consequence of the uncertainty relation. Contrary to this, Jetzer and Straumann [12] have recently expressed the view that the linear growth in the measured spectrum is produced by van der Waals forces and that it has nothing to do with zero-point fluctuations. But their view seems to be at variance with the standard view of almost all experts in the field of quantum noise theory (see [8] and references therein). The standard view is that the linear term in the measured spectrum is indeed a fingerprint of zero-point fluctuations of the harmonic oscillators which model the microscopic structure of the resistive element. Van der Waals forces, as well as the Casimir effect, are different effects that are relevant for other types of experimental situations (see, e.g. [13]), not for the noise in Josephson junctions.

Since zero-point fluctuations produce experimentally measurable effects in Josephson junctions, it is natural to conjecture that the energy density associated with the underlying primary fluctuations has physical meaning as well: It is a prime candidate for dark energy, being isotropically distributed and temperature independent. There is vacuum energy associated with the measured data in Fig. 1, and it cannot be easily discussed away.

Of course, one has to be careful in the interpretation of the experimental data in Fig. 1. What is really measured with the Josephson junction are real currents produced by real electrons. These currents behave very much in a classical way, however, they are *induced* by zero-point fluctuations. What type of zero-point fluctuations induce the measured currents in the first place is not clear from a theoretical point of view. It could be just ordinary vacuum fluctuations (of QED type), but there is also the possibility of other, new types of vacuum fluctuations, which could potentially underly dark energy and produce a measurable effect in the Josephson junction. What is clear is that if there were no zero-point fluctuations in the first place, then there would also be no linear term in the measured spectrum. Moreover, if the vacuum fluctuations inducing the noise in the junction had a cutoff at some frequency

 $\nu_{max}$ , and would fall quiet above that, the same effect would be observed for the induced physical noise spectrum. In any case, if such a cutoff exists then it would be the *new physics* underlying the cutoff mechanism that makes the system couple to gravity and thus make the corresponding vacuum energy physically relevant. This is the basic idea underlying the recent suggestion of laboratory tests on dark energy —checking whether there is a cutoff or not in an extended new version of the Koch experiment [7, 14].

### IV. ESTIMATING A COSMOLOGICAL CUTOFF FREQUENCY

Already Planck [15] and Nernst [16] formally included zero-point terms in their work. Their formulas yield for the energy density of a collection of oscillators of frequency  $\nu$ 

$$\rho(\nu,T) = \frac{8\pi\nu^2}{c^3} \left[ \frac{1}{2}h\nu + \frac{h\nu}{\exp(h\nu/kT) - 1} \right]$$
$$= \frac{4\pi h\nu^3}{c^3} \coth\left(\frac{h\nu}{2kT}\right). \tag{4}$$

This just corresponds to the measured spectrum in the Josephson junction experiment, up to a prefactor depending on R. We may split eq. (4) as

$$\rho(\nu, T) = \rho_{vac}(\nu) + \rho_{rad}(\nu, T), \tag{5}$$

where

$$\rho_{vac}(\nu) = \frac{4\pi h \nu^3}{c^3} \tag{6}$$

is induced by zero-point fluctuations, and

$$\rho_{rad}(\nu, T) = \frac{8\pi h \nu^3}{c^3} \frac{1}{\exp(h\nu/kT) - 1}.$$
 (7)

corresponds to the radiation energy density generated by photons of energy  $h\nu$ . Integration of eq. (6) up to some cutoff  $\nu_c$  yields

$$\int_0^{\nu_c} \rho_{vac}(\nu) d\nu = \frac{4\pi h}{c^3} \int_0^{\nu_c} \nu^3 d\nu = \frac{\pi h}{c^3} \nu_c^4.$$
 (8)

This is equivalent to eq. (1) with m=0. Integration of eq. (7) over all frequencies yields the well-known Stefan-Boltzmann law

$$\int_{0}^{\infty} \rho_{rad}(\nu, T) d\nu = \frac{\pi^2 k^4}{15\hbar^3 c^3} T^4. \tag{9}$$

Assuming that vacuum fluctuations (of whatever type) are responsible for dark energy, the necessary cutoff frequency follows from the current astronomical estimates of dark energy density [1, 2]:

$$\rho_{dark} = 0.73 \rho_c = (3.9 \pm 0.4) \text{ GeV/m}^3$$
 (10)

Here  $\rho_c$  denotes the critical density of a flat universe. From

$$\frac{\pi h}{c^3} \nu_c^4 \simeq \rho_{dark} \tag{11}$$

we obtain

$$\nu_c \simeq (1.69 \pm 0.05) \times 10^{12} \text{ Hz.}$$
 (12)

So if vacuum fluctuations underly dark energy, and if these vacuum fluctuations drive the corresponding quantum oscillators of frequency  $\nu$  in the resistive element, one would expect to see a cutoff near  $\nu_c$  in the measured spectrum of the Josephson junction experiment. Because otherwise the corresponding vacuum energy density would exceed the currently measured dark energy density. The frequency  $\nu_c$  is about 3 times higher than the largest frequency reached

in Koch et al.'s experiment of 1982. Future experiments, based on new Josephson junction technology, will be able to reach this higher frequency (see last section).

Suppose a cutoff near  $\nu_c$  is observed in a future experiment. Then this would represent new physics and the consequences would be far reaching. Frampton [17] has shown that the observation of such a cutoff could even shake some of the assumptions underlying string theory, leading to the possible demise of the so-called string landscape. His argument is based on the fact that if the cutoff is seen in the Josephson experiment then this implies that the dark energy field interacts with the electromagnetic field—which leads to a potential problem for string theory if the physical vacuum, as it is usually assumed, decays by a 1st order phase transition: A small cosmological constant as generated in this context would have decayed to zero by now, contradicting the fact that we do see dark energy right now. In any case, it is interesting that for the first time there seems to be an experiment that can check some of the assumptions underlying the string landscape.

#### V. NEW TYPES OF VACUUM FLUCTUATIONS?

As said before: If we allow for the possibility of new physics then there are many possible models of dark energy. Let us here consider the possibility that dark energy is produced by new types of vacuum fluctuations with a suitable cutoff. The model was introduced in detail in [18], here we just sketch the main idea.

We start from a homogeneous self-interacting scalar field  $\varphi$  with potential  $V(\varphi)$ . Most dark energy models are just formulated in a classical setting, but for our approach in terms of vacuum fluctuations we need of course to proceed to a second-quantized theory. We second-quantize our scalar field using the Parisi-Wu approach of stochastic quantization [19], which is a convenient and useful method for our approach. The second-quantized field  $\varphi$  then obeys a stochastic differential equation of the form

$$\frac{\partial}{\partial s}\varphi = \ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) + L(s,t),\tag{13}$$

where H is the Hubble parameter, t is physical time, s is fictitious time (just a formal coordinate to do quantization) and L(s,t) is Gaussian white noise,  $\delta$ -correlated both in s and t. The fictitious time s is introduced as a formal tool for 2nd quantization, it has dimensions  $GeV^{-2}$ . Quantum mechanical expectations can be calculated as expectations of the above stochastic process for  $s \to \infty$ . The advantage of the Parisi-Wu approach is that it is very easy and natural to introduce cutoffs in this formulation — by far easier than in the canonical field quantization approach. The simplest way to introduce a cutoff is by making t and t discrete (as in any numerical simulation of a stochastic process). Hence we write

$$s = n\tau \tag{14}$$

$$t = i\delta, \tag{15}$$

where n and i are integers and  $\tau$  is a fictitious time lattice constant,  $\delta$  is a physical time lattice constant. Note that the uncertainty relation  $\Delta E \Delta t = O(\hbar)$  always implies an effective lattice constant  $\Delta t$  for a given finite energy  $\Delta E$ . We also introduce a dimensionless field variable  $\Phi_n^i$  by writing  $\varphi_n^i = \Phi_n^i p_{max}$ , where  $p_{max}$  is some (so far) arbitrary energy scale. The above scalar field dynamics is equivalent to a discrete dynamical system of the form

$$\Phi_{n+1}^{i} = (1 - \alpha)T(\Phi_{n}^{i}) + \frac{3}{2}H\delta\alpha(\Phi_{n}^{i} - \Phi_{n}^{i-1}) + \frac{\alpha}{2}(\Phi_{n}^{i+1} + \Phi_{n}^{i-1}) + \tau \cdot noise, \tag{16}$$

where the local map T is given by

$$T(\Phi) = \Phi + \frac{\tau}{p_{max}(1-\alpha)}V'(p_{max}\Phi)$$
(17)

and  $\alpha$  is defined by

$$\alpha := \frac{2\tau}{\delta^2}.\tag{18}$$

For old universes, one can neglect the term proportional to H, obtaining

$$\Phi_{n+1}^{i} = (1 - \alpha)T(\Phi_{n}^{i}) + \frac{\alpha}{2}(\Phi_{n}^{i+1} + \Phi_{n}^{i-1}) + \tau \cdot noise$$
(19)

We now want to construct a field that basically manifests itself as noise: Rather than evolving smoothly it should exhibit strongly fluctuating behavior, so that we may be able to interpret its rapidly fluctuating behaviour in terms of

vacuum fluctuations, and possibly in terms of measurable noise in the Josephson junction. As a distinguished example of a  $\varphi^4$ -theory generating such behaviour, let us consider the map

$$\Phi_{n+1} = T_{-3}(\Phi_n) = -4\Phi_n^3 + 3\Phi_n \tag{20}$$

on the interval  $\Phi \in [-1, 1]$ .  $T_{-3}$  is the negative third-order Tchebyscheff map, a standard example of a map exhibiting strongly chaotic behaviour. It is conjugated to a Bernoulli shift, thus generating the strongest possible chaotic behaviour possible for a smooth low-dimensional deterministic dynamical system [20]. The corresponding potential is given by

$$V_{-3}(\varphi) = \frac{1-\alpha}{\tau} \left\{ \varphi^2 - \frac{1}{p_{max}^2} \varphi^4 \right\} + const, \tag{21}$$

or, in terms of the dimensionless field  $\Phi$ ,

$$V_{-3}(\varphi) = \frac{1 - \alpha}{\tau} p_{max}^2 (\Phi^2 - \Phi^4) + const.$$
 (22)

The important point is that starting from this potential we obtain by second quantization a field  $\varphi$  that rapidly fluctuates on some finite interval, choosing initially  $\varphi_0 \in [-p_{max}, p_{max}]$ . Since these chaotic fluctuations are bounded, there is a natural cutoff.

The idea is now that the expectation of the potential of this chaotic field (plus possibly kinetic terms) underly the measured dark energy density in the universe. Expectations  $\langle \cdots \rangle$  can be easily numerically determined by iterating the dynamics (19) for random initial conditions. One has

$$\langle V_{-3}(\varphi)\rangle = \frac{1-\alpha}{\tau} p_{max}^2 (\langle \Phi^2 \rangle - \langle \Phi^4 \rangle) + const,$$
 (23)

which for  $\alpha = 0$  can be analytically evaluated [20] to give

$$\langle V_{-3}(\varphi)\rangle = \frac{1}{8} \frac{p_{max}^2}{\tau} + const.$$
 (24)

To reproduce the currently measured dark energy, we only need to fix the ratio of the parameters  $\tau$  and  $p_{max}$  as

$$\frac{p_{max}^2}{\tau} \sim \rho_{\Lambda} \sim m_{\nu}^4 \tag{25}$$

This is the simplest model of noise-like vacuum fluctuations with a suitable finite cutoff one can think of, a chaotic scalar field theory underlying the cosmological constant. It is easy to show [18] that for  $\alpha = 0$  the equation of state of this field is w = -1. For small  $\alpha$ , it is close to w = -1. In this model, there is no reason why the deterministic chaotic noise represented by this rapidly fluctuating field should not be able to influence electric charges. Hence these chaotic fluctuations may well induce a measurable noise spectrum in Josephson junctions.

## VI. A NEW QUANTUM NOISE EXPERIMENT

In [7] we suggested to repeat the experiment of Koch et al. with new types of Josephson junctions that are capable of reaching higher frequencies. This new experiment will now be built, the grant has just been allocated [21]. Warburton, Barber, and Blamire are planning two different versions of the experiment. One is based on nitride junctions, the other one on cuprate junctions. The maximum frequency that can be reached is determined by the gap energy of the Josephson junction under consideration, and the above materials provide the possibility to reach the cosmologically interesting frequency of  $\nu_c \approx 1.7$  THz and even exceed it. The new technology suggested by Warburton et al. is more sophisticated than that of conventional Niobium-based Josephson junctions. With conventional Niobium-based junctions one would probably be only able to reach 1.5 THz in the measurements since their gap energy is too small. By performing experiments on both the nitrides and the cuprates there will be two independent high frequency measurements of the quantum noise spectrum in two very different material systems. So in about 3 years time we should know whether there is any unusual behaviour of zero-point fluctuations near  $\nu_c$ , which could possibly be related to dark energy. Whatever the outcome of this new experiment, the result will be interesting: If a cutoff is observed

it will revolutionize our understanding of dark energy. If a cutoff is not observed, it will show that the vacuum fluctuations measured with the Josephson junction are definitely not gravitationally active.

- [1] Bennett, C.L. et al. (2003), Astrophys. J. Supp. Series 148, 1 (astro-ph/0302207)
- [2] Spergel, D.N. et al. (2003), Astrophys. J. Supp. Series 148, 148 (astro-ph/0302209)
- [3] Peebles, P.J.E. and Ratra, B. (2003), Rev. Mod. Phys. 75, 559 (astro-ph/0207347)
- [4] Weinberg, S. (2000), astro-ph/0005265
- [5] Trodden, M. and Caroll, S. (2004), astro-ph/0401547
- [6] Padmanabhan, T. (2003), Phys. Rep. **380**, 235 (hep-th/0212290)
- [7] Beck, C. and Mackey, M.C., Phys. Lett. B **605**, 295 (2005) (astro-ph/0406504)
- [8] Gardiner, C.W. (1991), Quantum Noise, Springer, Berlin
- [9] Koch, R.H., van Harlingen, D. and Clarke J. (1980), Phys. Rev. Lett. 45, 2132
- [10] Koch, R.H., van Harlingen, D. and Clarke J. (1982), Phys. Rev. B 26, 74
- [11] Levinson, Y. (2003), Phys. Rev. B 67, 184504
- [12] Jetzer, P. and Straumann, N., Phys. Lett. B 606, 77 (2005) (astro-ph/0411034)
- [13] Bressi, G. et al., Phys. Rev. Lett. 88, 041804 (2002)
- [14] Ball, P., Nature **430**, 126 (2004)
- [15] Planck, M. (1914), The Theory of Heat Radiation. P. Blakistons's Son & Co.
- [16] Nernst, W. (1916), Verh. Dtsch. Phys. Ges., 18, 83
- [17] Frampton, P.H., hep-th/0508082
- [18] Beck, C. (2004) Phys. Rev. D 69, 123515 (astro-ph/0310479)
- [19] Parisi, G. and Wu, Y.S, Sci. Sin 24, 483 (1981)
- [20] Beck C., Spatio-temporal Chaos and Vacuum Fluctuations of Quantized Fields, World Scientific, Singapore (2002)
- [21] Warburton, P.A., Barber, Z., and Blamire, M., Externally-shunted high-gap Josephson junctions: Design, fabrication and noise measurements, EPSRC grants EP/D029783/1 and EP/D029872/1