INVARIANT MEASURES ON THE LATTICES OF THE SUBGROUPS OF THE GROUP AND REPRESENTATION THEORY *CONFERENCE DEVOTED TO PETER CAMERON, QUEEN MARY COLLEGE, LONDON, 8-10 JULY 2013*

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July 29, 2013

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Reference: A.V. Totally non-free actions and adjoint invariant measures for infinite symmetric group Moscow Math. J. 12, No. 1, 193-212 (2012). http://www.pdmi.ras.ru/ vershik/finenonfree.pdf

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I will give solution (V2009) of that Problem for infinite symmetric group $\mathfrak{S}_\infty$.

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3.Algebraic geometry of symmetric spaces (M. Abert, Y. Glasner and B. Virag. et al — "Invariant random subgroups" (IRS).

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1. *The map $\Psi : X \to L(G); x \mapsto Stab_x$ ("characteristic map") is a* mod0 *isomorphism* mod0 *of the action of G on (X, μ) and adjoint action $ad(G); H \mapsto H^g = g^{-1}Hg$ on $(L(G), \Psi_* \mu$.*
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("Different points have different stabilizers")
2. *The Boolean algebra $\mathfrak{B}$ generated by the family of the sets of the fixed points: $X_g = \{x : gx = x\}$ is complete Boolean algebra: $\mathfrak{B} =< X_g : g \in G >= \mathfrak{A}(X)$*

DYNAMICS ON THE LATTICE OF THE SUBGROUPS.

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Lemma

A transitive action of a group G (the left action of G on a homogeneous space G/H) is totally non-free if and only if all the stabilizers (i.e., the subgroup H) is a self-normalizing subgroup ($N(H) = H$, or $H \in LN(G)$).

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Denote as $LN(G) = \{H \in L(G) : N(H) = H\}$

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If a group G acts totally non free on the space (X, μ) then the image $\Psi_*\mu$ of the measure μ under the characteristic map $\Psi : x \to L(G)$, is concentrated on the set of self-normalizers :$(\Psi_*\mu)(LN) = 1$.

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We call such measures as TNF-measures.

The filtration of the normalizers

The filtration of the lattice of the subgroups is as follow:

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Observation: Each subgroup H of the group G can be imbedded to the uniquely defined self-normalizer subgroup $N\bar{N}$ which is limit of tower (transfinite in general) of the subgroups $N^\tau(H)$. Remark that the ordinal in the sequences of normalizers could be an arbitrary ordinal.

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The corresponding filtration of the measures used the following operation ("normalization"):

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We have

$$TNF = \{\mu \in AD : \mathcal{N}(\mu) = \mu\} \subset RTNF = \{\mu \in AD : \mathcal{N}(\mu) \in TNF\} =$$

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Remark The AD-measures on $(L(G)$, whose normalization is TNF we call "reduced TNF or $RTNF$-measures. Adjoint action of G on $(L(G), \mu)$ is TNF off μ is $RTNF$-measure.

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Consider a sequence of positive numbers $\alpha = \{\alpha_i\}_{i=0,1\ldots}$ such that

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Define a partition δ of indices $\mathbb{N} = \{i = 1, \dots\}$ onto finite or infinite number of blocks of three types subsets: $i \in P_+$, $i \in P_+$, P_-, $i \in P_c$, where each block of set of type P_+ and P_- consist with one point, and each block of type P_c consist with more than two points; of course it is possible that only one of the sets of blocks of type P_+, P_-, P_c is nonempty. Denote the block of partition δ which contains i as C_i (it is single-point block if $i \in P_+, P_-$).

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Main result

Consider the space of all sequences $\mathcal{N} = \mathbb{N}^{\mathbb{N}} = \{\{\xi_n\} : x_n \in \mathbb{N}, n \in \mathbb{N}\}$ and consider Bernoulli measure M_α with probability vector $\alpha = \{\alpha_i\}; i \in \mathbb{N}; \alpha_i > 0$: $Pr(\{\xi : \xi_n = i\}) = \alpha_i$ and $\{\xi_n\}$ are i.i.d. We have Bernoulli space $(\mathcal{N}, M_\alpha)$. Let $\{\xi_n\}; n \in \mathbb{N}$ is random sequence from corresponding Bernoulli ensemble

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Let $i \in P_c$, and C_i is block of partition δ which contains i, then group G_i is the subgroup of the product: $\prod \mathfrak{S}_{\cup N_j, j \in C_i}$, of the elements of type $(g_1, g_2 \dots)$ all of which have the same parity; this subgroup depends on the block C_i (not of individual i).

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Theorem

For each ergodic AD-measure μ on the lattice of the subgroups of the group $\mathfrak{S}_\mathbb{N}$ there exists a unique sequence $\alpha_i, i \in \mathbb{N}$ and partition δ of above type of $\mathbb{N}$, such that a map $\xi \mapsto G^\xi$ is isomorphism mod0 *between Bernoulli space $(\mathcal{N}, M_\alpha)$ and $(L(\mathfrak{S}_\mathbb{N}), \mu)$.*

We called such a subgroups as random signed Young subgroup.

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2.AD-measure can't concentrated on the class of intransitive subgroups.
3.Each ergodic AD-measure concentrated on the subgroups "like product of symmetric or alternation subgroups.

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or more general formula:

$$\chi(g) = \alpha(g, H)\mu(\{H : g \in H\}),$$

jj where $\alpha(g, H)$ is a $\pm$-cocycle.

Theorem

For infinite symmetric group $\mathfrak{S}_\infty$ this type of the characters exhausts all of the characters

It is not clear for what groups the is formula for all characters. F.e. not true for finite groups.

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Usual analog of transitivity topological or (everywhere density of all orbit of G) or measure-theoretic = ergodicity no invariant sets of intermediate measure.).

Definition

1. We say that the action of a group of measure preserving transformations is it metrically k-transitive if for almost every (in the sense of the measure μ^k on X^k) choice of points $x_1, x_2, \dots, x_{k-1}$, the action of the intersection of subgroups stabilizers $\bigcap_{i=1}^{k-1} G_{x_i}$ on (X, μ) is ergodic.

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2. We say that the action of a group measure preserving transformations is *oligomorphic* if for any positive integer k and for almost all k-tuple points $x_i, i = 1, 2 \ldots k$ the number of ergodic components of the intersection of stabilizers $\bigcap_{i=1}^{k-1} G_{x_i}$ on (X, μ) is finite.

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Problem For what groups there exist k-transitive actions for given k? oligomorphic action? When adjoint action with respect to AD-measures is k-transitive? oligomorphic?

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Problem To describe the list of AD-invariant measures for the group $GL(F_q, \infty)$, and $GLB(F_q)$.
Conjecture: the AD-problem for those groups is smooth.
Problem In what sense the problem of the description of AD-invariant measures for free groups F_k is universal in the class of all countable groups.

HAPPY BIRTHDAY, DEAR PETER! HUGE SET OF THE PROBLEMS FROM ALL MATH!