Combinatorics, Algebra, and More: A Conference in Celebration of Peter Cameron

Abstracts (updated 09 July 2013)

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László Babai

(University of Chicago, USA.)

Symmetry Versus Regularity.

The theme of this talk is the long-recognised seeming paradox that regularity constraints often impose severe limitations on the number and structure of symmetries, the archetype of such results being the fact that doubly transitive permutation groups, other than the alternating and the symmetric groups, are tiny: their orders are quasipolynomially bounded (less than exponential of a polynomial of the logarithm; in this case, a quadratic polynomial).

In the talk, I will report brand new developments in an area that connects some of Peter Cameron's favourite topics, including finite geometries, strongly regular graphs, primitive permutation groups, and his recent favourite, combinatorial relaxations to the notion of a base of a permutation group. The questions are partly motivated by a problem Peter posed a quarter-century ago in connection with his study of oligomorphic groups.

The new results give a quasipolynomial bound on the number of automorphisms of Steiner t-designs and give new bounds and asymptotic structural constraints on the automorphism groups of strongly regular graphs. One of the results asserts that if X is a strongly regular graph with automorphism group G then, with known (trivial) exceptions, G has a subgroup of quasipolynomial index that is a Γ_μ group, i.e., a group of which every composition factor is a subgroup of the symmetric group of degree μ. (μ denotes the number of common neighbours of a pair of non-adjacent vertices of X.) An interesting 'clique geometry' is also found in strongly regular graphs in certain parameter ranges.

The results are joint work with my student John Wilmes and partly simultaneous and partly joint work with Xi Chen, Xiaorui Sun, and Shang-Hua Teng.

Martin Liebeck

(Imperial College London, UK.)

Fixed points of elements of permutation groups and linear groups.

R. A. Bailey

(Queen Mary, University of London, UK / University of St Andrews, UK.)

Incomplete-block designs and Laplacian eigenvalues.

If an experiment must be conducted in blocks, and there is not room in each block for every treatment, then an incomplete-block design must be used. How do we choose which design is best for the given parameters? We want the variance to be small, but if there are more than two treatments, then the variance is a multi-dimensional function and there are different ways of getting a single summary number to minimize. The most common of these are functions of the Laplacian eigenvalues of two graphs defined by the incomplete-block design: its *Levi graph* and its *concurrence graph*.

I shall talk about ongoing joint work with PJC.

Robert Johnson

(Queen Mary, University of London, UK.)

Optimal resistor networks.

It is possible to think of a graph as an electrical network by replacing each edge with a 1 ohm resistor. This viewpoint has applications to some diverse areas of mathematics including random walks, partitioning rectangles into squares, and statistical design theory.

A statistical application motivates the extremal problem of minimising the average resistance between pairs of vertices, over all graphs with a given number of edges. We will discuss some new results and open questions related to this problem.

This is joint work with Mark Walters.

Peter Cameron

(Queen Mary, University of London, UK / University of St Andrews, UK.)

Primitivity.

Throughout mathematics, we find ourselves needing to 'reduce' objects to simpler pieces which cannot be further reduced. In permutation groups, it is traditional to take the simple pieces as the primitive groups.

In my talk, I will present eight different conditions which are equivalent to primitivity, taken from many areas (automata theory, experimental design, and graph theory, among others). Often, it turns out that these properties can be quantified in different ways, or have modifications or extensions to the infinite which are no longer the same as primitivity.

On the way we see quite a bit of recent work on synchronization, transformation semigroups, graph homomorphisms, and some of my other current interests.

Geoff Whittle

(Victoria University of Wellington, NZ.)

Partial Fields and Matroid Representation.

In matroid representation theory, it is natural to attempt to characterize the matroids representable over all members of a given set of fields. For example, regular matroids are the matroids representable over all fields and these are possibly the most intensively studied class of matroids. Some time ago, Semple and Whittle observed that in all known cases, such classes of matroids could be described as matroids representable over a certain algebraic structure that they termed a 'partial field'. Thus motivated, they began the formal study of partial fields. Vertigan soon obtained a number of interesting results, but then things languished until partial fields were revived by van Zwam in his recent PhD thesis. In my talk, I will develop the basics of the theory and present some of the more interesting results. I will also present some open problems — the area is replete with these. I chose to talk on this topic because I believe that the interplay of algebraic and geometric concepts is reflective of themes that have played an ongoing role in Peter's own research.

Donald A. Preece

(Queen Mary, University of London, UK / University of Kent, UK.)

Tredoku tilings.

We have a supply of identical tiles, each in the shape of a rhombus with alternate angles 60^o and 120^o, so each tile has 4 sides and 4 vertices. We use τ of these tiles ($\tau > 4$) to form a tiling $\mathcal{T}$ of part of a plane. In $\mathcal{T}$, the position of any vertex of a tile is a vertex of $\mathcal{T}$, and the position of any side of a tile is a side of $\mathcal{T}$.

To be a *tredoku tiling*, $\mathcal{T}$ must have these properties:

1. $\mathcal{T}$ is connected, with no connection consisting merely of two tiles touching at just one vertex;
2. If two tiles in $\mathcal{T}$ touch, either they do so only at a vertex or they have a side in common in $\mathcal{T}$;
3. $\mathcal{T}$ cannot be disconnected by removing just one tile (but a connection consisting merely of two tiles touching at just one vertex is allowed after the removal);
4. If tiles A and B share a side p of $\mathcal{T}$, then there is a third tile C that shares a side q of either A or B, where q is parallel to p. This gives a "run" of three tiles.
5. Runs of more than three tiles are not allowed.
6. $\mathcal{T}$ must not have any holes in it.

(There is no requirement for any part of the tiling to repeat.) If a tredoku tiling has ρ runs of three tiles, then

$$\frac{3}{2}\rho \leq \tau \leq 2\rho + 1\,.$$

Many existence results and constructions are available for tredoku tilings. Motivation: If each tile in $\mathcal{T}$ is subdivided into a 3×3 array of smaller tiles, we have an overall grid for a tredoku© puzzle such as appears daily in *The Times*.

João Araújo

(Universidade Aberta, Portugal / Centro de Álgebra da Universidade de Lisboa, Portugal.)

The Wall

To study an object (for example, the natural numbers), we start by identifying a relevant subobject (prime numbers) which in some sense 'controls' the behaviour of the whole object. In semigroup theory, there are two obvious candidates to be taken as this relevant subobject: the group of units or the set of idempotents.

Fifty years ago, while idempotents were poorly understood, groups were (and remain) among the most studied of all classes of algebras. Therefore, prima facie, all bets should be placed on the group of units.

There was only one small detail: the nature of the questions which semigroup theorists had to ask to group theorists. For example:

1. Is it possible (for a fixed k) to classify the groups such that the orbit of any k-set contains a section of each k-partition?

2. Is it possible to classify the degree n primitive groups of diameter at most n?

Fifty years ago, questions of this type were simply a brick wall. So, for decades, experts in semigroup theory turned to the study of idempotents. In this talk, I am going to show what happened to that wall on the 25th of June, 2010, from 9:45 to 11:05.

Alan Sokal

(New York University, USA / University College London, UK.)

Some wonderful conjectures (but very few theorems) concerning the leading root of some formal power series.

Many problems in combinatorics, statistical mechanics, number theory and analysis give rise to power series (whether formal or convergent) of the form

$$f(x,y) \;=\; \sum_{n=0}^{\infty} a_n(y)\, x^n \;,$$

where $\{a_n(y)\}$ are formal power series or analytic functions satisfying $a_n(0) \neq 0$ for $n = 0, 1$ and $a_n(0) = 0$ for $n \geq 2$. Furthermore, an important role is played in some of these problems by the roots $x_k(y)$ of $f(x, y)$ — especially the 'leading root' $x_0(y)$, i.e. the root that is of order y^0 when $y \to 0$. Among the interesting series $f(x, y)$ of this type are the 'partial theta function'

$$\Theta_0(x, y) \;=\; \sum_{n=0}^{\infty} x^n \, y^{n(n-1)/2} \;,$$

which arises in the theory of q-series, and the 'deformed exponential function'

$$F(x, y) \;=\; \sum_{n=0}^{\infty} \frac{x^n}{n!} \, y^{n(n-1)/2} \;,$$

which arises in the enumeration of connected graphs. These two functions can also be embedded in natural hypergeometric and q-hypergeometric families.

In this talk I will describe recent (and mostly unpublished) work concerning these problems — work that lies on the boundary between analysis, combinatorics and probability. In addition to explaining my (very few) theorems, I will also describe some amazing conjectures that I have verified numerically to high order but have not yet succeeded in proving. My hope is that one of you will succeed where I have not!

Thomas Prellberg

(Queen Mary, University of London, UK.)

The Combinatorics of the leading root of Ramanujan's function.

I consider the leading root $x_0(q)$ of Ramanujan's function,

$$\sum_{n=0}^{\infty} \frac{(-x)^n q^{n^2}}{(1-q)(1-q^2)\ldots(1-q^n)}.$$

I prove that its formal power series expansion,

$$q x_0(-q) = 1 + q + q^2 + 2q^3 + 4q^4 + 8q^5 + \ldots,$$

has positive integer-valued coefficients, by giving an explicit combinatorial interpretation of these numbers in terms of trees whose vertices are decorated with polyominos. Similar results are obtained for the leading roots of the partial Theta function and the Painleve Airy function.

Mark Jerrum

(Queen Mary, University of London, UK.)

Graph homomorphisms and generalisations: a computational perspective.

A homomorphism from one graph G to another H is a mapping from $V(G)$ to $V(H)$ that sends edges of G to edges of H. Fix the graph H and consider the problem of deciding whether there exists a homomorphism from G to H. Around 1990, Hell and Nešetřil investigated the computational complexity of this decision problem and proved a dichotomy theorem: except in two trivial cases, namely when H has a loop or H is bipartite, the decision problem is NP-complete. Hell and Nešetřil's work sparked a major research effort into potential variants and generalisations of their result. In the last few years, progress in this direction has been rapid, though the most important conjecture — that the dichotomy can be extended from homomorphisms of graphs to homomorphisms of general structures — remains open. I'll cover a selection of recent results, concluding with some recent work with Leslie Goldberg (Oxford) on the complexity of estimating, within specified relative error, the number of homomophisms to a fixed graph H, in the special case when H is a tree.

Dugald Macpherson

(University of Leeds, UK.)

Maximal-closed permutation groups and reducts of structures.

In a 1976 paper which has stimulated a huge amount of mathematics, Peter Cameron showed that any infinite permutation group which is highly homogeneous but not highly transitive, either 'preserves' or 'preserves or reverses' a linear or circular order. In particular, if S is the quaternary 'separation relation' on the set $\mathbb{Q}$ of rational numbers, derived from the natural circular ordering, then $\mathrm{Aut}(\mathbb{Q}, S)$ is maximal in $\mathrm{Sym}(\mathbb{Q})$ subject to being closed in the topology of pointwise convergence, and the ω-categorical first order structure $(\mathbb{Q}, S)$ has no proper non-trivial 'reducts'.

Since then, the reducts of many homogeneous first order structures (or equivalently, the closed supergroups of the automorphism group) have been classified; this work has recently been stimulated by connections to constraint satisfaction problems. Junker and Ziegler asked whether there are any countable structures which are not ω-categorical but which have just finitely many reducts. In this setting, the group-theoretic and model-theoretic notions of reduct differ. I will describe joint work with Manuel Bodirsky, giving an example of a countable structure which is not ω-categorical but has no proper non-trivial reduct, and whose automorphism group is maximal-closed in the symmetric group.

Bridget Webb

(The Open University, UK.)

Infinite designs - a quick tour.

Although examples of infinite designs from geometry have been around for a long while, the full definition of an infinite design is relatively recent (2002). In this quick tour, I will give a variety of examples, and then outline some methods of construction, concentrating on infinite Steiner systems, to demonstrate the breadth of the subject.

Anatoly Vershik

(Steklov Institute of Mathematics, Russia / St. Petersburg State University, Russia.)

Random subgroups of groups, and applications to representation theory.

We study the conjugation-invariant probability measures on the lattice of subgroups of the infinite symmetric group (and of various other groups). We apply this to the theory of factor representations, and also to the classification of the actions of these groups.

Colva Roney-Dougal

(University of St Andrews, UK.)

Random games with finite groups.

Peter Neumann

(University of Oxford, UK.)

The Rev. Thos. P. Kirkman, M.A., F.R.S. and his Complete Theory of Groups.

This contribution to the conference will be about the context, content and significance of Kirkman's articles on groups published between 1861 and 1863.