MAS224, Actuarial Mathematics: Problem Sheet 8

Post your solutions to the starred questions in the ***orange box*** *on the* ***second floor*** *of the Maths building by* ***12 noon on Monday, 31st March 2008****. Do not forget to staple all pages together and write your name and student number at the top of the front sheet.*

Assume 4% interest, ignore expenses and (where appropriate) use the approximation for the whole life annuities due paid p-thly or continuously derived in lectures. In questions 1 and 3 assume select values from the A1967-70 tables. Give your numerical answers to the nearest penny.

1*. A man on retirement on his 65th birthday receives a lump sum of $£100,000$ which he uses to purchase a pension which will be paid monthly in advance. Find the monthly pension he receives.

2*. Joe Bloggs wins a prize which can be taken either as a lump sum payment of $£10,000$ paid immediately or as monthly payments of $£100$ per month paid in advance for life. Joe is aged 60 and his mortality is given in table A1967-70 ultimate values. Find which form of prize is preferable and justify your answer.

3*. (a) John Doe takes out a whole-life assurance on his 50th birthday with a sum assured of £20,000. Find the annual premium for this assurance if premiums are to start immediately and to be paid on each birthday.

(b) Suppose that the John Dow actually died at the age of 60, so that the death benefit was paid on (what would have been) his 61st birthday. Calculate the actual loss made by the life assurance company on John Doe's policy at the time when the benefit was paid.

(c) Suppose that John Dow actually survived to the age of 60. Find the surrender value of his policy at that time.

4*. (a) Consider an n-year temporary life annuity-due with monthly payments at rate 1 p.a. for a life aged x now, i.e. a level annuity-due, contingent on the survival of (x), payable monthly in advance at rate 1 p.a. for at most of n years (the maximum number of payments possible is $12n$). Denote by $\ddot{a}^{(12)}_{x:\overline{n}|}$ its expected present value. Show that

$$\ddot{a}^{(12)}_{x:\overline{n}|} = \frac{1}{12} \sum_{j=0}^{12n-1} \frac{D_{x+\frac{j}{12}}}{D_x}$$

(b) Consider an n-year term endowment policy such that (i) it provides for a benefit either on the death of (x) or on the survival of (x) to the end of the n-year term whichever event occurs first; (ii) the death benefit is payable at the end of the month of death; and (iii) the death and survival benefits are both of 1 unit of money.

Let Z_1 and Z_2 be the present values of the death and survival benefits, correspondingly. Write these two random variables in terms of $T(x)$, and hence find their probability mass functions. Let $A^{(12)}_{x:\overline{n}|}$ be the expected present value of the benefits under this policy, so that $A^{(12)}_{x:\overline{n}|} = E(Z_1) + E(Z_2)$. Show that

$$A^{(12)}_{x:\overline{n}|} = 1 - d^{(12)}\, \ddot{a}^{(12)}_{x:\overline{n}|}.$$