Actuarial Mathematics Test 2007

Write your name and student number below. Answer all questions

Student Number: Name:

Table ELT12 is provided on the last page and present values of annuities-certain are given below:

$$\ddot{a}_{\overline{n}|} = \frac{1-V^n}{1-V} \qquad a_{\overline{n}|} = V\,\ddot{a}_{\overline{n}|}$$

$$\ddot{a}^{(p)}_{\overline{n}|} = \frac{1}{p}\left[\frac{1-V^n}{1-V^{\frac{1}{p}}}\right] \qquad a^{(p)}_{\overline{n}|} = V^{\frac{1}{p}}\,\ddot{a}^{(p)}_{\overline{n}|}.$$

1. Interest is compounded monthly. The monthly rate of interest is 1%,

   (i) Find the annual equivalent rate of interest; *(5 marks)*

   (ii) Find the annual equivalent rate of discount. *(5 marks)*

   (iii) How much interest should be paid in arrears for the use of £5,000 over a one year period? *(3 marks)*

   (iv) How much interest should be paid in advance for the use of £5,000 over a one year period? *(3 marks)*

2. Regular savings of £100 per month are made in advance for 3 years. The APR paid on the account is 5% per annum. Find the accumulation at the end of the 3 year period. *(12 marks)*

3. John Brown is repaying a loan of £10,000 by annual payments in arrears over a 10 year period. The interest rate is variable and initially the APR charged is 8%. After 3 years the APR is increased to 10% so that his annual payment has to be increased in line with the new interest rate. The term of the loan remains unaltered.

   (i) Find his initial repayment level.

   (ii) Find the amount of loan outstanding at the time of the interest rate change, i.e. when his third payment has just been made.

   (iii) Find the new repayment level.

4. The lifetime X has density function $f_X(x) = \frac{1}{5}$ for $0 < x < 5$.

   (i) Find the survival function $S(x)$ for $0 \leq x \leq 5$. *(2 marks)*

   (ii) If $K(x)$ is the curtate further lifetime of a life aged x, write $P(K(x) = k)$ in terms of the survival function and hence calculate $P(K(x) = k)$ and specify the range. *(8 marks)*

   Calculate $\mathsf{e}_x = E[K(x)]$ for $x = 0, 1, 2, 3, 4$. *(10 marks)*

5 Use the life table ELT12 to obtain the following results:

(a) Calculate the probability that a life aged 20 survives to age 65. *(6 marks)*

(b) Calculate the expected number of deaths by age 40 out of 1000 30-year olds. *(6 marks)*

(c) Show that ${}_{n|t}q_x = {}_np_x \times {}_tq_{x+n}$. *(4 marks)*

(d) Let x be an integer and let $0 < t < 1$. Use linear interpolation of the survival function between integer ages to show that ${}_tq_x = t \times q_x$. *(6 marks)*

(e) Find the probability that a life who has just reached his 65^{th} birthday survives to 70 but dies within the next month. *(8 marks)*