Queen Mary
University of London

B. Sc. Examination by course unit 2009

MTH4108 Probability I

Duration: 2 hours

Date and time: 13 August 2009, 14:30

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): J Robert Johnson

Question 1 [12]

Let E and F be events.

(a) Write down the following events in symbols:

(i) F does not occur,

(ii) E and F both occur.

(b) Write down the following events in words:

(i) $E \setminus F$,

(ii) $E^c \cap F^c$.

(c) Write down the inclusion-exclusion formula for $\mathbb{P}(E \cup F)$.

Question 2 [11]

A race takes place between Amanda, Boris and Carl. The order in which the three runners finish is recorded.

(a) Write down the sample space for this experiment.

(b) Write down the following events as subsets of your sample space:

(i) "Amanda finishes ahead of Boris"

(ii) "The race is won by Carl"

(iii) "Boris does not finish last"

(c) Another race takes place between n runners rather than 3. Write down the cardinality of the sample space for this experiment.

Question 3 [10]

Let A and B be two events with $\mathbb{P}(A) > 0$ and $\mathbb{P}(B) > 0$.

(a) What does it mean for A and B to be independent?

(b) What does it mean for A and B to be disjoint?

(c) Show that if A and B are independent then they are not disjoint.

(d) Show that if A and B are disjoint then $\mathbb{P}(A) + \mathbb{P}(B) \leq 1$.

Question 4 [12]

Let X be a discrete random variable with probability mass function

n	0	1	2	3
$P(X=n)$	$1/10$	$3/10$	$1/2$	$1/10$

Calculate the following:

(a) $\mathbb{P}(X>1)$,

(b) $\mathbb{E}(X)$,

(c) $\text{Var}(X)$,

(d) $\mathbb{P}(X^2+2>5)$.

Question 5 [9]

(a) Write down the probability mass function (pmf) of a $\text{Bin}(n,p)$ random variable.

(b) Give an example of a practical situation which is modelled by a $\text{Bin}(3,1/4)$ random variable.

(c) Suppose that $B\sim\text{Bin}(3,1/4)$ Calculate the following probabilities:

(i) $\mathbb{P}(B=2)$,

(ii) $\mathbb{P}(B \text{ is even})$.

Question 6 [6]

Let S be the right-angled triangle with vertices $(0,0),(1,0)$ and $(1,1)$. Let a be a point chosen at random from within the triangle with the probability that a is in any fixed region being proportional to the area of the region. Let X be the random variable "the x-coordinate of a". Find the probability density function (pdf) of X.

Question 7 [20]

(a) Say what it means for a function $f : X \to Y$ to be *injective*.

(b) Write down an example of function $f : \mathbb{Z} \to \mathbb{Z}$ which is injective.

(c) Write down an example of function $f : \mathbb{Z} \to \mathbb{Z}$ which is not injective.

Let $g : \{1,2\} \to \{1,2,3\}$ be a randomly chosen function with all possibilities equally likely.

(d) Find the probability that $g(1) < g(2)$.

(e) Find the probability that g is injective.

(f) Are the events $g(1) = 1$ and $g(2) = 2$ independent? Justify your answer.

Let $h : \{1,2\} \to \{1,2,3\}$ be a randomly chosen injective function with all possibilities equally likely.

(g) Are the events $h(1) = 1$ and $h(2) = 2$ independent? Justify your answer.

Question 8 [20]

(a) State and prove Bayes' theorem.

(b) Under what circumstances do we have $\mathbb{P}(A|B) > \mathbb{P}(B|A)$?

A certain disease is carried by $1/100$ proportion of a population. If a person with the disease is tested for it there is probability $9/10$ that the test shows that they have the disease. If a person without the disease is tested for it there is probability $1/9$ that the test shows they have the disease. A randomly chosen person is tested for the disease.

(c) Calculate the probability that the test shows they have the disease.

(d) Calculate the conditional probability that they have the disease given that the test shows that they do.

(e) Calculate the probability that the test correctly diagnoses whether or not they have the disease.

End of Paper