

## MTH4108 Probability 1 – 2009/10 – Exercise Sheet 7

*These questions are designed to help you understand the Theorem of Total Probability and Bayes' Theorem. You should write up your solution to the starred question,  $Q4^*$ , clearly and hand it in during your week 9 exercise class for feedback. Put your **full name and student number** on the top of your solution. It is important that you make a serious attempt to do **all** of questions  $Q1$ - $Q4$  before week 9 lectures begin. Questions  $AQ1$ - $AQ2$  are for additional practice. You should attempt them when you have time.*

Q1. (a) Which of the following partition  $S$  when  $A$  and  $B$  are arbitrary events:

- (i) the four events  $A, A^c, B, B^c$
- (ii) the four events  $A \setminus B, B \setminus A, A \cap B, (A \cup B)^c$
- (iii) the three events  $A \cap B, A \Delta B, A^c \cap B^c$

(b) Write down any partition of  $S = \{1, 2, 3, \dots, 10\}$  into 3 non-empty events  $E_1, E_2, E_3$ .

Q2. In the 2008 US presidential election 66% of voters aged under 30 voted for Barack Obama. Also, Obama's share of the total vote was 52.6% and 18% of all voters were aged under 30.

- (i) What is the probability that a voter aged 30 or over voted for Obama?
- (ii) What is the probability that someone who voted for Obama was aged under 30?

Q3. Two important players in a football team are injured, and each has probability  $1/3$  of recovering before the next match. The recoveries of the two players are independent of each other. If both are able to play then the team has probability  $3/4$  of winning the match, if only one of them plays then the probability of winning is  $1/2$  and if neither play the probability of winning is  $1/16$ . What is the probability that the team wins the match?

Q4\*. I have three coins in my pocket: two of them are ordinary fair coins; the third has a head on both sides.

- (i) I take a coin out of my pocket at random and toss it. What is the probability that it comes down heads?
- (ii) Suppose that I take a coin out of my pocket at random, toss it, and it does come down heads. What is the probability that the other side is a head?
- (iii) What is the probability that if I toss the same coin a second time I will get another head?

AQ1. A student observes that out of all students graduating with first class degrees in mathematics from Queen Mary , 96% of them passed the mid-term test in Probability I. He claims that since he has just passed the test he has a very good chance of getting a first.

- (i) What is the most obvious mistake with this argument? Explain your answer in terms of conditional probabilities.
- (ii) Assume further that 80% of students pass the test and 10% of students get a first. What is the probability that a randomly chosen student who has passed the test gets a first.

AQ2. In a game of tennis once the score has reached deuce play continues until (effectively) one player has a lead of two points.

The score has reached deuce in a game of tennis between Andre and Boris. Suppose that each point is won by Andre with probability  $1/4$  (and otherwise by Boris) and that the outcome of each point is independent of all other points.

Let  $x$  be the probability that Andre wins,  $u$  be the conditional probability that Andre wins given that he wins the first point and  $v$  be the conditional probability that Andre wins given that he loses the first point.

Use the theorem of total probability to show that

$$4x = u + 3v$$

$$4u = 3x + 1$$

$$4v = x$$

and hence determine the probability that Andre wins the game. (Hint. Let  $W_1$  be the event that Andre wins the first point and  $W_2$  be the event that Andre wins the second point. Apply Theorem 9.3 using the partition  $W_1, W_1^c$  and Theorem 9.4 using the partition  $W_2, W_2^c$ .)

Comment briefly on what the relation between your answer and the probability that Andre wins each point implies about the game of tennis.