

## MTH4108 Probability 1 – 2009/10 – Exercise Sheet 6

*These questions are designed to help you understand the material covered in week 6 lectures. You should write up your solution to the starred question, Q3\*, clearly and hand it in during your week 8 exercise class for feedback. Put your **full name and student number** on the top of your solution. It is important that you make a serious attempt to do **all** of questions Q1-Q3 before week 8 lectures begin. Questions AQ1-AQ2 are for additional practice. You should attempt them when you have time.*

Q1. A couple keep having children until they either have two girls or have four children (as in question Q1 on exercise sheet 5). Assume that they do not have twins, or other multiple births and that each child they have is equally likely to be a boy or a girl independently of all other children.

- (i) Find the probability that they have two girls.
- (ii) Find the conditional probability that they have two girls given that their first child is a boy.
- (iii) Find the conditional probability that their first child is a boy given that they have two girls.
- (iv) Find the conditional probability that their first child is a girl given that they have two girls.

Q2. Let  $A$  and  $B$  be events with  $\mathbb{P}(A), \mathbb{P}(B) > 0$ .

- (i) Show that  $\mathbb{P}(A^c|B) = 1 - \mathbb{P}(A|B)$ .
- (ii) Show that if  $\mathbb{P}(A|B) < \mathbb{P}(A)$  then  $\mathbb{P}(B|A) < \mathbb{P}(B)$ .
- (iii) Illustrate both these results using question Q1.

Q3\*. Two treatments for a disease are tested on a group of 390 patients. Treatment  $A$  is given to 160 patients of whom 100 are men and 60 are women; 20 of these men and 40 of these women recover. Treatment  $B$  is given to 230 patients of whom 210 are men and 20 are women; 50 of these men and 15 of these women recover.

- (i) For which of  $A$  and  $B$  is there a higher probability that a patient chosen randomly from among those given that treatment recovers? Express this as an inequality between two conditional probabilities.
- (ii) For which of  $A$  and  $B$  is there a higher probability that a man chosen randomly from among those given that treatment recovers? Express this as an inequality between two conditional probabilities.
- (iii) For which of  $A$  and  $B$  is there a higher probability that a woman chosen randomly from among those given that treatment recovers? Express this as an inequality between two conditional probabilities.
- (iv) Are you surprised by your answers to the previous parts? What is going on here?

AQ1. When I travel into work each morning I notice whether my train is late and by how much and also whether I am able to get a seat on it. Let  $A$  be the event “the train is on time”,  $B$  be the event “the train is late but by not more than 15 minutes”, and  $C$  be the event “I am able to get a seat”. Suppose that  $\mathbb{P}(A) = 1/2$ ,  $\mathbb{P}(B) = 1/4$ ,  $\mathbb{P}(C) = 1/3$  and  $\mathbb{P}(A \cap C) = 1/4$ .

- (i) Find the conditional probability that the train is more than 15 minutes late given that the train is late.
- (ii) Find the conditional probability that I get a seat given that the train is late.

AQ2. Prove that for any events  $A$ ,  $B$  and  $E$  with  $\mathbb{P}(E) > 0$  we have

$$\mathbb{P}(A \cup B|E) = \mathbb{P}(A|E) + \mathbb{P}(B|E) - \mathbb{P}(A \cap B|E).$$