

MTH4108 Probability 1 – 2009/10

Exercise Sheet 5

These questions are designed to help you understand the material covered in week 5 lectures. You should write up your solution to the starred question, Q1, clearly and hand it in during your week 6 exercise class for feedback. Put your **full name and student number** on the top of your solution. It is important that you make a serious attempt to do **all** of questions Q1-Q5 before week 6 lectures begin. Questions AQ1-AQ4 are for additional practice. You should attempt them when you have time.*

Q1*. A couple wants two girls. They decide to keep having children and stop either when they have two girls or when they have four children, whichever happens first. Assume that they do not have twins, or other multiple births and that each child they have is equally likely to be a boy or a girl independently of all other children.

- (i) Write down the sample space.
- (ii) Are all elements of the sample space equally likely? Justify your answer.
- (iii) Calculate the probability that they have two girls.

Q2. The top card of a thoroughly shuffled deck of playing cards¹ is turned over. Let A be the event “the card is an Ace”, R be the event “the card belongs to a red suit (\diamond or \heartsuit)”, and M be the event “the card belongs to a major suit (\heartsuit or \spadesuit)”. Show that the events A , R and M are mutually independent.

Q3. A positive integer from the set $\{1, 2, 3, \dots, 36\}$ is chosen at random with all choices equally likely. Let E be the event “the chosen number is even”, O be the event “the chosen number is odd”, and Q be the event “the chosen number is a perfect square”.²

- (i) Are the events E and O independent? Justify your answer.
- (ii) Are the events E and Q independent? Justify your answer.
- (iii) Are the events O and Q independent? Justify your answer.

Q4. There are two roads from A to B and two roads from B to C . Suppose that each road is closed with probability p and that the state of each road is independent of the others. What condition on p ensures that the probability that I cannot travel from A to C is no more than $1/2$?

¹A deck of playing cards is made up of 52 cards split into 4 suits (\clubsuit , \diamond , \heartsuit , \spadesuit) with each suit made up of one card of each of 13 ranks (2, 3, 4, \dots , 10, Jack, Queen, King, Ace).

²An integer is a *perfect square* if it is equal to the square of an integer.

Q5. Let A and B be events. Prove that if A and B are independent then A^c and B are independent. Which question on this sheet illustrates a special case of this?

AQ1. You are allowed up to three attempts to pass the Probability I exam. Suppose that your probability of passing is p at each attempt, independent of all other attempts. How big must p be so that the probability that you eventually pass the exam is at least 0.9? Give your answer to 4 decimal places.

AQ2. You are at Stepney Green station and you want to get to Liverpool Street station. You can either go directly by the Hammersmith and City line, or you can take the District line to Mile End followed by the Central line to Liverpool Street. For simplicity, let us assume that there are no other Underground lines. The probability that the Hammersmith and City line is working is 0.7; the probability that the District line is working is 0.8; and the probability that the Central line is working is 0.9. Assume that each line is independent of the others. What is the probability that you can get to Liverpool Street by Underground train?

AQ3. Let A and B be independent events. Suppose also that the probability of A is twice the probability of B and that the probability that neither of A and B occur is $1/2$. Find $\mathbb{P}(A)$.

AQ4. A positive integer from the set $\{1, 2, 3, \dots, 60\}$ is chosen at random with all choices equally likely. For each positive integer k let M_k be the event “the chosen number is a multiple of k ”.

- (i) Are the events M_2, M_3 and M_5 mutually independent?
- (ii) Are the events M_2, M_4 and M_5 mutually independent?
- (iii) Formulate and answer a more general version of this question.