MTH4108 Probability 1 – 2009/10

Exercise Sheet 2

These questions are designed to help you understand the material covered in week 2 lectures. You should write up your solution to the starred question, Q2,* **clearly** *and hand it in during your week 3 exercise class for feedback. Put your* **full name and student number** *on the top of your solution. It is important that you make a serious attempt to do* **all** *of questions Q1-Q5 before week 3 lectures begin. Questions AQ1-AQ3 are for additional practice. You should attempt them when you have time.*

Q1. In each of the following cases say whether or not f defines a function from A to B. Give brief explanations.

(i) $A = \{1, 2, 3, 4\}, \quad B = \{1, 2, 3, 4\}, \quad f(x) = x - 1.$

(ii) $A = \{1, 2, 3, 4\}, \quad B = \{1, 2, 3, 4\}, \quad f(x) = 5 - x.$

(iii) $A = B = \mathbb{R}$,

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x > 1 \end{cases}$$

(iv) $A = B = \mathbb{R}$,

$$f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x \geq 1 \end{cases}$$

Q2*. For each of the following functions f say whether or not f is surjective, whether or not f is injective, and whether or not f is bijective. Give brief explanations. For those functions which have an inverse say what the inverse function is. (You are asked lots of things about each function in this question; make sure you don't miss any of them out!)

(i) $f : \mathbb{N} \to \mathbb{N}, \quad f : x \mapsto 3x + 4$

(ii) $f : \mathbb{R} \to \mathbb{R}, \quad f : x \mapsto 3x + 4$

(iii) $f : \mathbb{R} \to \{x \in \mathbb{R} : -1 \leq x \leq 1\}, \quad f : x \mapsto \sin(x)$

(iv) $f : \mathbb{R} \to \mathbb{R}, \quad f : x \mapsto \begin{cases} -x^2 & \text{if } x \leq 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

Q3. Let L be the set of lecturers in a department and C be the set of courses taught by the department. It is suggested that the allocation of lecturers to courses is described by a function $t : C \to L$ where t maps a course to whoever teaches it.

(i) What condition on the teaching allocation is required in order that t be a function?

(ii) Assuming that t is a function, say in words what it means for it to be injective.

(iii) Assuming that t is a function, say in words what it means for it to be surjective.

Q4.

(i) Find a function $f : \mathbb{N} \to \mathbb{N}$ which is injective but not surjective.

(ii) Find a function $f : \mathbb{N} \to \mathbb{N}$ which is surjective but not injective.

(iii) Let A be a finite set. Is it possible to find a function $f : A \to A$ which is injective but not surjective, or surjective but not injective? Why?

Q5. Find the cardinalities of the following sets:

(i) The set of all ordered triples (x, y, z) with $x, y, z \in \{0, 1\}$.

(ii) The set of all functions from $\{1, 2, 3\}$ to $\{0, 1\}$.

What is the connection between the two parts of this question?

AQ1. For each of the following either give an example of such a function or say why there is no such function:

i) an injective function $f : \{1, 2, 3, 4\} \to \{1, 2, 3\}$,

ii) a surjective function $f : \{1, 2, 3, 4\} \to \{1, 2, 3\}$,

iii) an injective function $f : \{1, 2, 3\} \to \{1, 2, 3, 4\}$,

iv) a surjective function $f : \{1, 2, 3\} \to \{1, 2, 3, 4\}$.

AQ2. Let A, B, C be finite sets. Show that if $f : A \to B$ and $g : B \to C$ are both bijections then $g \circ f : A \to C$ is a bijection.

AQ3. Give an example of a bijective function from $\mathbb{N}$ to $\mathbb{Z}$.