MTH4108 Probability I – 2009/10

Solutions to Exercise Sheet 1

Q1.

(i) If the pair ij denotes that the first roll is number i and the second roll is number j then the sample space is

$$\begin{aligned}\{&11, 12, 13, 14, 15, 16,\\ &21, 22, 23, 24, 25, 26,\\ &31, 32, 33, 34, 35, 36,\\ &41, 42, 43, 44, 45, 46,\\ &51, 52, 53, 54, 55, 56,\\ &61, 62, 63, 64, 65, 66\}\end{aligned}$$

You could also write this as

$$\{(x, y) : x, y \in \mathbb{N}, 1 \leq x \leq 6, 1 \leq y \leq 6\}$$

or (using the notation for Cartesian product) as

$$\{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}.$$

(ii) The sample space has 36 elements. (Either just count them or note that there are 6 possibilities for the first roll and for each of these there are 6 possibilities for the second. Hence, there are $6 \times 6 = 36$ possibilities in total).

Q2*.

(i)

$$S = \{hh, hth, htth, httt, thh, thth, thtt, tthh, ttht, ttt\}$$

(ii) This event is

$$\{htth, httt, thth, thtt, tthh, ttht\}$$

(iii) Two elements of the sample space are $ttthh$ and $htttttttttth$. Of course there are many other choices. Any sequence which ends with a tail and contains at least two heads and exactly three tails will do, as will any sequence which ends with a head and contains at least three tails and exactly two heads.

In contrast to part (i) this sample space is infinite. For instance, there are infinitely many sequences of the form k heads followed by 3 tails in S. Another difference is the sample space contains some infinite sequences.

Q3.

(i) The simplest way to express S is to list the horses which finish in the order in which they finish (so CA means C finishes first, A finishes second, B falls). We record the outcome that all horses fall as X. You may have chosen a different notation which is fine provided that you explained it. The sample space is

$$\{X, A, B, C, AB, BA, AC, CA, BC, CB, ABC, ACB, BAC, BCA, CAB, CBA\}.$$

(ii) With the notation above this event is

$$\{A, AB, AC, ABC, ACB\}.$$

(iii) With the notation above this event is

$$\{X, A, C, AC, CA\}.$$

(iv) With the notation above this event is

$$\{ABC, ACB, BAC, BCA, CAB, CBA\}.$$

Q4.

(i) $A \cap B = \{3, 4\}$, $|A \cap B| = 2$.

(ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $|A \cup C| = 8$.

(iii) $B \triangle C = \{3, 4, 7, 8\}$, $|B \triangle C| = 4$.

Q5.

(i) This is true. The set $D_2 \cap D_3$ is the set of all positive integers which are divisble by both 2 and 3. Any number which is divisible by 2 and 3 must have 2 and 3 appearing in its factorisation into primes and so is divisible by 6. Also any number which is divisible by 6 is certainly divisible by 2 and 3. It follows that the set of positive integers which are divisible by 2 and 3 is the same as the set of positive integers which are divisible by 6. In other words $D_2 \cap D_3 = D_6$.

(ii) This is false since $40 \in D_8$.

(iii) This is true. If an integer is divisible by 4 then it is certainly divisible by 2 and so every element of D_4 is also an element of D_2. In other words D_4 is a subset of D_2.

(iv) This is true. If $n \in D_5^c$ then $n \notin D_5$ (by definition) that is n is not divisible by 5. But if n is not divisible by 5 then it is also not divisible by 15 and so $n \notin D_{15}$. It follows that if $n \in D_5^c$ then $n \notin D_{15}$. In other words D_5^c and D_{15} are disjoint.

AQ3(i) $\emptyset$
(ii) $\mathbb{Z} \setminus \{0\}$
(iii) $\mathbb{Z}_+ \cup \{0\}$ (or $\mathbb{N}$)
(iv) $\{2\}$
(v) $\mathbb{Z}_+ \setminus \{2\}$

Please let me know if you have any comments or corrections