MTH4108 Probability I – 2009/10

Exercise Sheet 1

These questions are designed to help you understand the material covered in week 1 lectures. You should write up your solution to the starred question, Q2,* **clearly** *and hand it in during your week 2 exercise class for feedback. Put your* **full name and student number** *on the top of your solution. It is important that you make a serious attempt to do* **all** *of questions Q1-Q5 before week 2 lectures begin. Questions AQ1-AQ3 are for additional practice. You should attempt them when you have time.*

Q1. You throw an ordinary six-sided die, whose faces bear the numbers $1,\ldots,6$, and note what number is showing. Then you throw it again, and note what number is showing. (So you have written down two numbers.)

(i) Write down the sample space, explaining your notation carefully.

(ii) How many elements are there in the sample space?

Q2*. You toss an ordinary coin repeatedly, recording the outcome of each toss. You do this until you have seen either two Heads or three Tails in total and then you stop.

(i) Write down the sample space.

(ii) Write down the event "you toss the coin exactly four times" as a subset of the sample space.

I perform the same experiment but I do not stop until I have seen both two Heads and three Tails in total.

(iii) Write down any two elements of the sample space for my experiment. Comment on one major difference between the sample space for my experiment and your answer to part (i).

Q3. A race takes place between three horses A, B and C. It is possible that one or more of them may fall and so fail to complete the race. The finishing horses are recorded in the order in which they finish.

(i) Write down the sample space, explaining your notation carefully.

(ii) Write down the event "The race is won by horse A" as a subset of the sample space.

(iii) Write down the event "Horse B falls" as a subset of the sample space.

(iv) Write down the event "All horses complete the race" as a subset of the sample space.

Q4. Let

$$A = \{1,2,3,4\}, \quad B = \{3,4,5,6\}, \quad C = \{5,6,7,8\}.$$

Find the following sets and their cardinalities:

$$\text{(i)} \quad A \cap B; \quad \text{(ii)} \quad A \cup C; \quad \text{(iii)} \quad B \triangle C.$$

Q5. Let $\mathbb{Z}_+$ be the set of all positive integers. For each $k \in \mathbb{Z}_+$, let D_k be the subset of $\mathbb{Z}_+$ containing those integers which are divisible by k. Decide whether the following statements are true or false giving a brief explanation in each case:

(i) $D_2 \cap D_3 = D_6$,

(ii) $40 \in (D_4 \cap D_5) \setminus D_8$,

(iii) $D_4 \subset D_2$,

(iv) D_5^c and D_{15} are disjoint.

AQ1. Four students, Amanda, Brian, Claire and David, are taking a lecture course. In the last lecture of the semester it is noted which of them are present.

(i) Write down the sample space, explaining your notation carefully.

(ii) Write down the event "Exactly three of them attend the last lecture" as a subset of the sample space.

(iii) Write down the event "More than half of them are absent from the last lecture" as a subset of the sample space.

(iv) Write down the event "Amanda attends the last lecture but David does not" as a subset of the sample space.

AQ2. Let

$$A = \{1,2,3,4,5\}, \quad B = \{4,5,6,7,8\}.$$

(i) Find a set $C \subset \mathbb{N}$ which satisfies $|C| = 5$, $|C \cap A| = |C \cap B| = 2$, and $|A \cup B \cup C| = 11$.

(ii) Find a set $D \subset \mathbb{N}$ which satisfies $|D| = 5$, $|D \cap A| = |D \cap B| = 2$, and $|A \cup B \cup D| = 9$.

(iii) Explain why there is no set $E \subset \mathbb{N}$ which satisfies $|E| = 5$, $|E \cap A| = |E \cap B| = 2$, and $|A \cup B \cup E| = 12$.

AQ3. Let $\mathbb{Z}$ be the set of all integers, $\mathbb{Z}_+$ be the set of positive integers, $\mathbb{Z}_-$ be the set of negative integers, O be the set of odd positive integers, and R be the set of prime numbers. Find the following sets:

$$\text{(i)} \quad \mathbb{Z}_- \cap \mathbb{Z}_+; \quad \text{(ii)} \quad \mathbb{Z}_- \cup \mathbb{Z}_+; \quad \text{(iii)} \quad \mathbb{Z} \setminus \mathbb{Z}_-; \quad \text{(iv)} \quad R \setminus O; \quad \text{(v)} \quad (\mathbb{Z}_+ \setminus O) \triangle (R \cup O).$$