

Probability I – 2009/10

Solutions to End-Semester Test

Q1.

- (i) Let E_1, E_2, \dots, E_m be events which partition a sample space S and satisfy $\mathbb{P}(E_i) > 0$ for all $1 \leq i \leq m$. Let A be an event in S . Then

$$\mathbb{P}(A) = \mathbb{P}(A|E_1)\mathbb{P}(E_1) + \mathbb{P}(A|E_2)\mathbb{P}(E_2) + \dots + \mathbb{P}(A|E_m)\mathbb{P}(E_m) \quad [10]$$

- (ii) Let W, D, L be the events that the teams wins, draws or loses the next game. Let S be the events that the manager is sacked. Then

$$\begin{aligned} \mathbb{P}(S) &= \mathbb{P}(S|W)\mathbb{P}(W) + \mathbb{P}(S|D)\mathbb{P}(D) + \mathbb{P}(S|L)\mathbb{P}(L) \\ &= \frac{1}{10} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{6} + \frac{9}{10} \times \frac{1}{2} \\ &= \frac{1}{30} + \frac{1}{12} + \frac{9}{20} = \frac{17}{30} \end{aligned} \quad [15]$$

Q2.

- (i)

$$\begin{array}{c|ccc} x & 0 & 1 & 2 \\ \hline P(X = x) & 1/4 & 1/4 & 1/2 \end{array} \quad [10]$$

- (ii)

$$\mathbb{E}(X) = 0 \times \frac{1}{4} + 1 \times \frac{1}{4} + 2 \times \frac{1}{2} = \frac{5}{4} \quad [5]$$

- (iii)

$$\mathbb{E}(X^2) = 0^2 \times \frac{1}{4} + 1^2 \times \frac{1}{4} + 2^2 \times \frac{1}{2} = \frac{9}{4}$$

So

$$\text{Var}(X) = \frac{9}{4} - \left(\frac{5}{4}\right)^2 = \frac{11}{16} \quad [10]$$

Q3.

(i) Throw a fair die three times and let X be the number of throws which are 5 or 6. (Solution not unique). [5]

(ii)

x	0	1	2	3
$P(X = x)$	$(2/3)^3$	$3(2/3)^2(1/3)$	$3(2/3)(1/3)^2$	$(1/3)^3$
$P(X = x)$	$8/27$	$12/27$	$6/27$	$1/27$

[10]

(iii)

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 8/27 & \text{if } 0 \leq x < 1 \\ 20/27 & \text{if } 1 \leq x < 2 \\ 26/27 & \text{if } 2 \leq x < 3 \\ 1 & \text{if } 3 \leq x \end{cases}$$

[10]

Q4.

(i)

$$f_X(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 3x^2 & \text{if } 0 < x < 1 \\ 0 & \text{if } 1 < x \end{cases}$$

[10]

(ii)

$$\mathbb{E}(X) = \int_0^1 3x^3 dx = [3x^4/4]_0^1 = 3/4$$

[5]

(iii)

$$\mathbb{E}(X^2) = \int_0^1 3x^4 dx = [3x^5/5]_0^1 = 3/5$$

So

$$\text{Var}(X) = 3/5 - (3/4)^2 = 3/80$$

[10]