

MAS108 Probability I

Mid-term Test

9:30am, Friday 10 November 2006

Duration: 45 minutes

Answer All Questions. Each question carries 20 marks.

Calculators are not permitted in this examination.

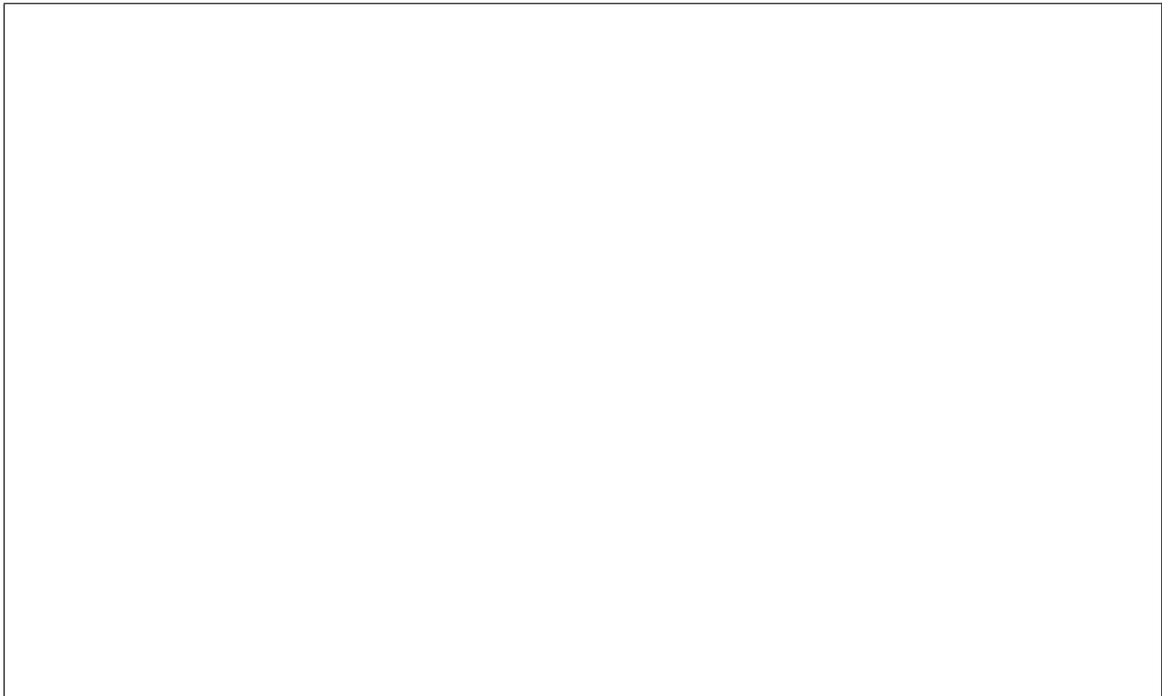
Write your answers on the question paper. If you need more space then use the blank pages at the end of the booklet and be sure to number your answers clearly

NAME:

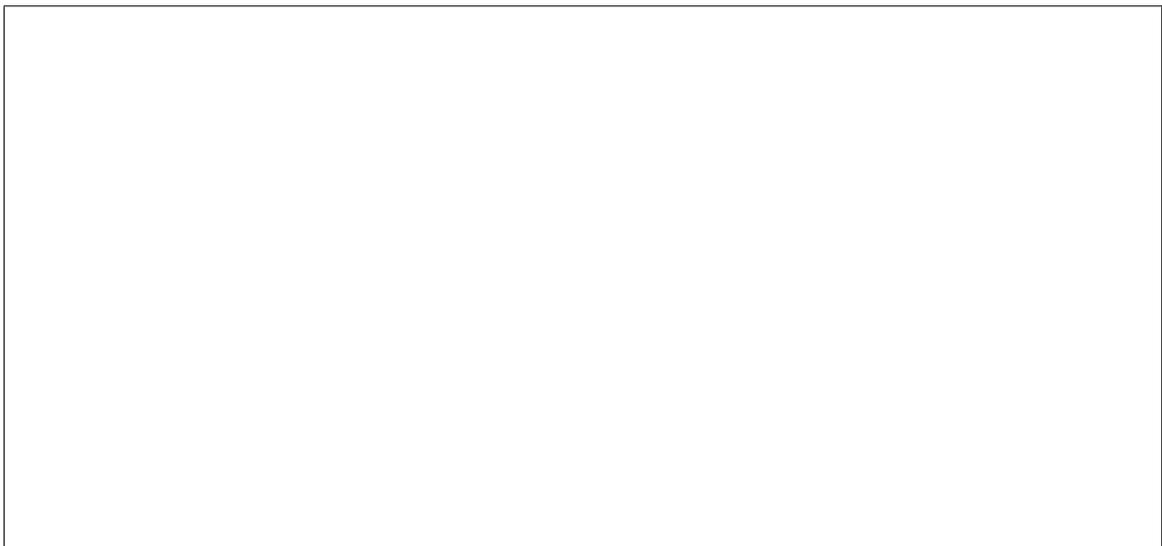
STUDENT NUMBER:

1. A 4-sided die with faces labelled by the numbers 1,2,3,4 is rolled and the number showing is recorded. Then an ordinary 6-sided die is rolled and the number showing is recorded. You may assume that both dice are fair and so each element of the sample space is equally likely. Let A be the event “the 6-sided die shows a 3”, and B be the event “the number showing on the 4-sided die is strictly greater than the number showing on the 6-sided die”.

- i) Write down the sample space and express the events A and B as subsets of the sample space.



- ii) Are A and B independent?

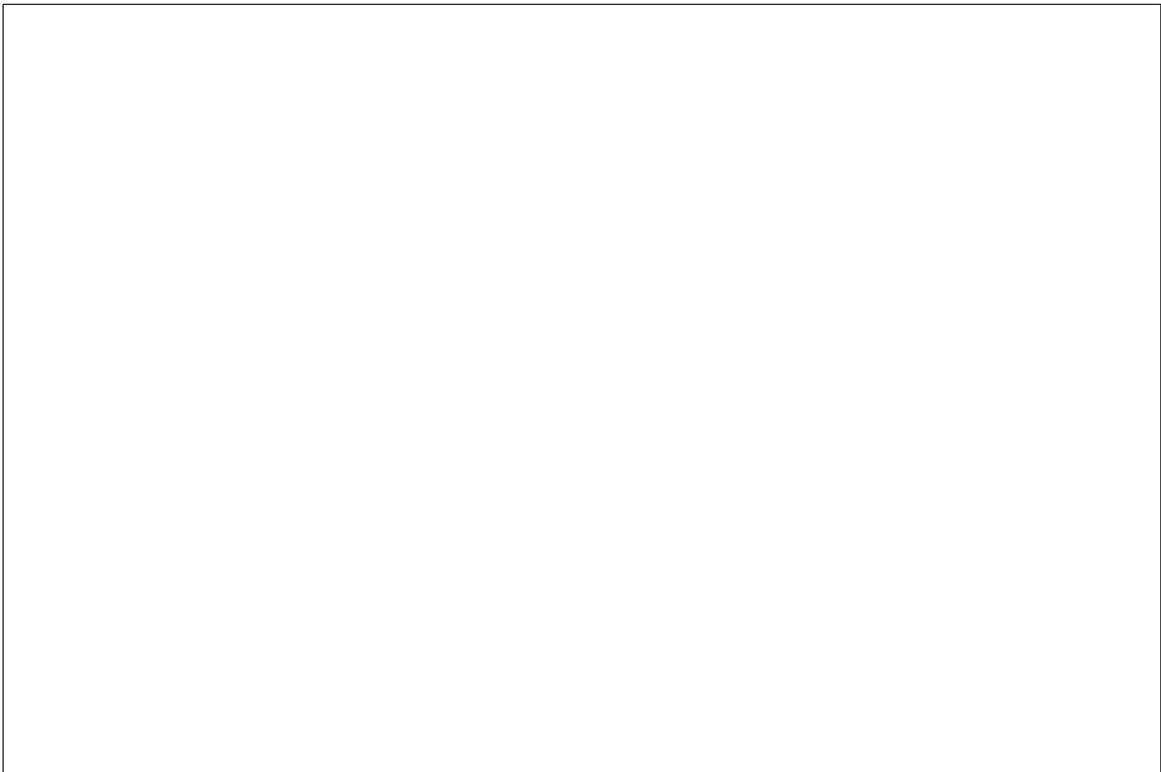


2.

i) State Kolmogorov's axioms for probability.



ii) Use the axioms to prove that if A and B are events with $A \subseteq B$ then $\mathbb{P}(A) \leq \mathbb{P}(B)$.

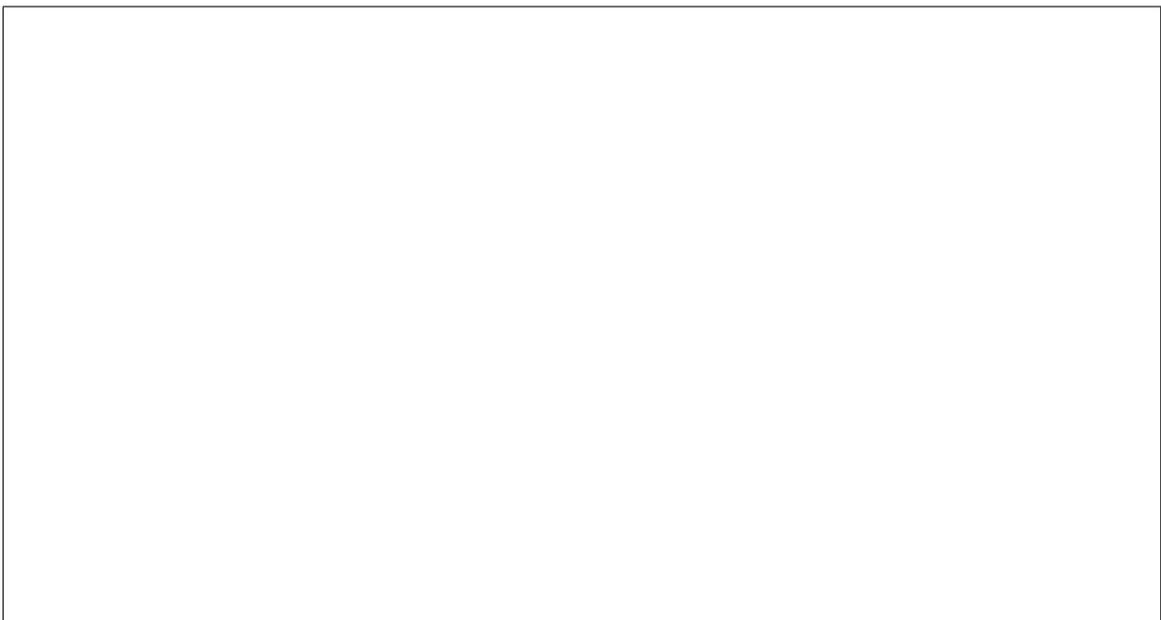


3. A squad of 8 five-a-side football players contains 2 goalkeepers and 6 other players. An eccentric manager chooses a team of 5 players randomly with all possibilities being equally likely.

- i) What is the probability that the team does not contain a goalkeeper? Express your answer first in terms of binomial coefficients, and then as a fraction.



- ii) What is the probability that the team contains exactly one goalkeeper? Express your answer first in terms of binomial coefficients, and then as a fraction.

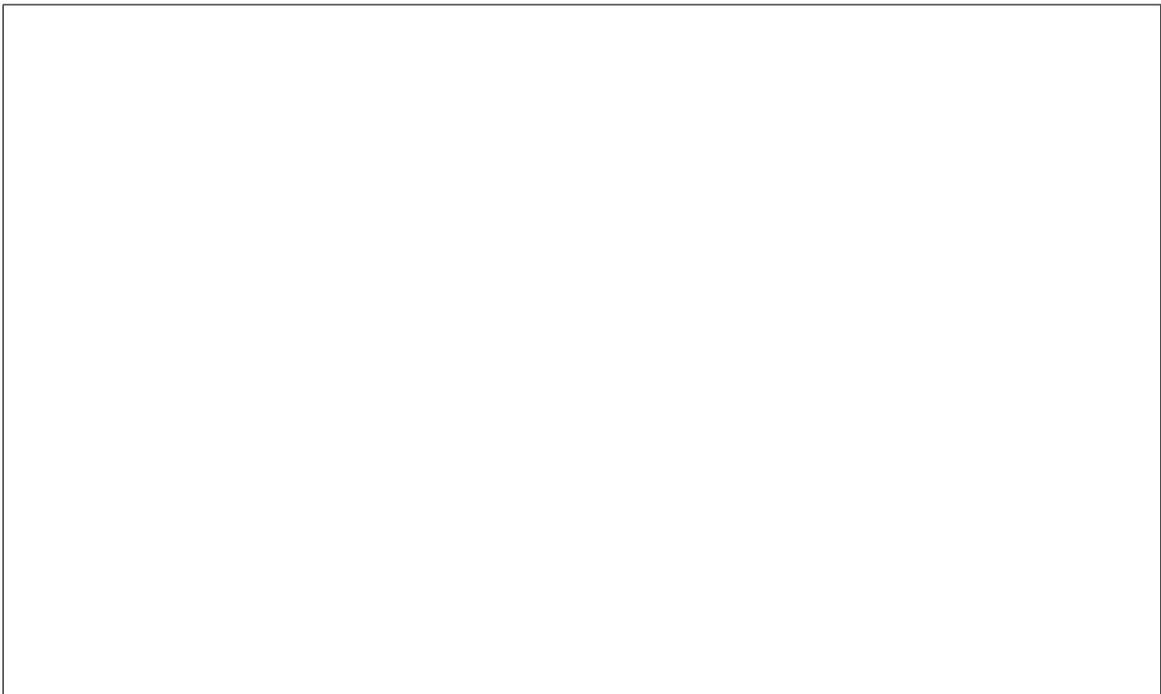


4. In a group of 100 students 60 study algebra, 80 study calculus, and 54 study both algebra and calculus. A student is chosen at random with all of the 100 equally likely to be chosen

i) What is the probability that the chosen student does not study either subject?



ii) What is the conditional probability that the chosen student studies algebra given that they study calculus.



5. Let A and B be events with $\mathbb{P}(B), \mathbb{P}(A) > 0$.

i) Define the conditional probability of A given B .

ii) Say what it means for A and B to be independent.

iii) What can you say about the conditional probability of A given B when A and B are independent? Prove your assertion.

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