

Solutions - exercise 3

1) $\langle M(t)^m \rangle = (M(0))^m e^{m\mu t + \frac{1}{2}\sigma^2 t m^2}$ (Q1, coursework 1)

$t < t_D \Rightarrow S(t) = M(t) \Rightarrow \langle S(t)^m \rangle = (S(0))^m e^{m\mu t + \frac{1}{2}\sigma^2 t m^2}$

$t \geq t_D \Rightarrow S(t) = (1-f) M(t) \Rightarrow \langle S(t)^m \rangle = (S(0))^m (1-f)^m e^{m\mu t + \frac{1}{2}\sigma^2 t m^2}$

2) Lecturers: $\frac{\partial C}{\partial K} = -e^{-rt} \phi(\omega - \sigma\sqrt{t})$

thus $\frac{\partial^2 C}{\partial K^2} = -e^{-rt} \phi'(\omega - \sigma\sqrt{t}) \cdot \frac{\partial \omega}{\partial K}$

$\omega = \frac{rt + \frac{1}{2}\sigma^2 t - \log K - \log S}{\sigma\sqrt{t}} \Rightarrow \frac{\partial \omega}{\partial K} = -\frac{1}{\sigma\sqrt{t}} \frac{1}{K}$

$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}y^2} dy \Rightarrow \phi'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

$\Rightarrow \frac{\partial^2 C}{\partial K^2} = \underbrace{e^{-rt}}_{>0} \frac{1}{\sqrt{2\pi}} \underbrace{e^{-\frac{1}{2}(\omega - \sigma\sqrt{t})^2}}_{>0} \cdot \underbrace{\frac{1}{K\sigma\sqrt{t}}}_{>0} > 0 \Rightarrow \text{convex}$

3) $\left. \begin{aligned} S + P_1 - C_1 &= Ke^{-rt} \\ S + P_2 - C_2 &= Ke^{-2rt} \end{aligned} \right\} \text{subtract} \Rightarrow P_1 - C_1 - P_2 + C_2 = K(e^{-rt} - e^{-2rt})$

(put-call option parity)

Let $x := e^{-rt} \Rightarrow \frac{P_1 - C_1 - P_2 + C_2}{K} = x - x^2$

$\Leftrightarrow x^2 - x + \frac{1}{K}(P_1 - C_1 - P_2 + C_2) = 0$

$x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{K}(P_1 - C_1 - P_2 + C_2)} = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{2}{100}} = \frac{1}{2} \pm \sqrt{\frac{23}{100}}$

$\Rightarrow r = -\log x = -\log\left(\frac{1}{2} + \sqrt{\frac{23}{100}}\right) = 0.0206 = 2.06\%$

↑ reasonable interest rate for + sign

4) * e.g. August 2007

closing prices 3235.13
(in pence) 1 Aug

3258.33
2 Aug

3223.84
3 Aug

3200.15
6 Aug

...

\Rightarrow 20 data points

$X_1^* = 0.00715$

$X_2^* = -0.01064$

$X_3^* = -0.00738$

...

$\bar{X}^* = \frac{1}{20} \sum_{i=1}^{20} X_i^* = 0.00039$

$\sigma = \sqrt{\frac{1}{2} \frac{1}{n-1} \sum_{i=1}^n (X_i^* - \bar{X}^*)^2} \approx 0.31$

$l = \frac{t}{n} \quad t \approx \frac{1}{12} \quad n = 20$

$\mu = \frac{1}{l} \bar{X}^* = 0.094$

$MSE(\hat{\sigma}^2) = \text{var} \hat{\sigma}^2 \approx \frac{2}{n-1} \sigma^4 \approx 1 \cdot 10^{-3}$

31-Aug-07	3,200.15	3,261.72	3,200.15	3,261.72	0	3,261.72
6-Aug-07	3,223.84	3,228.25	3,190.21	3,200.15	0	3,200.15
3-Aug-07	3,258.33	3,270.38	3,217.21	3,223.84	0	3,223.84
2-Aug-07	3,235.13	3,266.87	3,235.13	3,258.33	0	3,258.33
1-Aug-07	3,289.12	3,289.12	3,202.45	3,235.13	0	3,235.13

3260.48

$$X_4 = 0.01906$$

$$X_3 = -0.00738$$

$$X_2 = -0.01064$$

$$X_1 = 0.00715$$

* Close price adjusted for dividends and splits.

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Marking scheme

- printed list (10)
- list of X_i (20)
- μ (20)
- σ (25)
- MSE (25)

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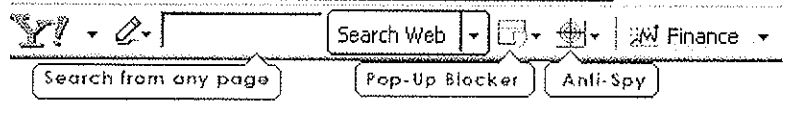
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$$\bar{X} = \frac{1}{20} \sum_{i=1}^{20} X_i = 0.00039$$

$$\text{sample variance} = \frac{1}{19} \sum_{i=1}^{20} (X_i - \bar{X})^2 = 4.02 \cdot 10^{-4}$$

$$l = \frac{t}{n} \quad t \approx \frac{1}{12} \quad (1 \text{ month})$$

$n = 20$
 ↑
 number of data points used

$$\Rightarrow l \approx 0.00417$$

$$\mu = \frac{1}{l} \bar{X} = 0.094$$

$$\sigma^2 \approx \frac{1}{l} \text{sample variance} = 0.0964 \Rightarrow \sigma \approx 0.31$$

$$MSE(\sigma^2) = \text{var } \sigma^2 \approx \frac{2}{n-1} \sigma^4 \approx 1 \cdot 10^{-3}$$

↑
lectures

This is an example only.

The numbers $\mu, \sigma, \text{var}(\sigma^2)$ may strongly fluctuate for the different months.