Solutions - exercise 2

1) $$r_i(j) = \begin{cases} \sigma_i^2 & \text{if } j = i \\ -3 & \text{if } j \neq i \end{cases}$$

Arbitrage theorem: $\exists\ p_1, \dots, p_m$ s.t.

$$0 = \sum_{j=1}^{m} p_j\, r_i(j) = p_i \sigma_i^2 - 3 \sum_{\substack{j=1 \\ j\neq i}}^{m} p_j = p_i \sigma_i^2 - 3(1-p_i)$$

$$\Rightarrow 3 = p_i \sigma_i^2 + 3p_i \Rightarrow p_i = \frac{3}{3+\sigma_i^2}$$

2) $$\frac{\partial C}{\partial r} = \left\langle I \cdot \frac{\partial}{\partial r}\{e^{-rt}(S(t)-K)\}\right\rangle \qquad \text{(lectures)}$$

$$\frac{\partial}{\partial r}\{e^{-rt}(S(t)-K)\} = -t e^{-rt}(S(t)-K) + e^{-rt}\frac{\partial}{\partial r} S(t)$$

$$S(t) = s \exp\{(r - \tfrac{1}{2}\sigma^2)t + \sigma\sqrt{t} Z\} \Rightarrow \frac{\partial}{\partial r} S(t) = t \cdot S(t)$$

$$\Rightarrow \frac{\partial}{\partial r}\{e^{-rt}(S(t)-K)\} = K t e^{-rt}$$

$$\Rightarrow \frac{\partial C}{\partial r} = \langle I \cdot K t e^{-rt}\rangle = K t e^{-rt}\langle I\rangle \underset{\text{lectures}}{=} K t e^{-rt}\phi(\omega - \sigma\sqrt{t})$$

3) Lemma 1 For geom. Brownian motion (risk-neutral)

$$S(t) = s \exp\{(r - \tfrac{1}{2}\sigma^2)t + \sigma\sqrt{t} Z\}$$

Z: Gaussian with mean 0, var 1

Lemma 2

$$I = \begin{cases} 1 & \text{if } Z > \sigma\sqrt{t} - \omega \\ 0 & \text{else} \end{cases} \qquad \text{where } \omega := \frac{rt + \frac{1}{2}\sigma^2 t - \log\frac{K}{s}}{\sigma\sqrt{t}}$$

indicator function of event that $S(t) > K$

Proof: $S(t) > K \Leftrightarrow s \exp\{(r - \tfrac{1}{2}\sigma^2)t + \sigma\sqrt{t} Z\} > K$ (Lemma 1)

$$\Leftrightarrow (r - \tfrac{1}{2}\sigma^2)t + \sigma\sqrt{t} Z > \log\frac{K}{s}$$

$$\Leftrightarrow Z > \frac{\log\frac{K}{s} - (r - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

Now $$\sigma\sqrt{t} - \omega = \frac{\sigma^2 t - rt - \frac{1}{2}\sigma^2 t + \log\frac{K}{s}}{\sigma\sqrt{t}} = \frac{\frac{1}{2}\sigma^2 t - rt + \log\frac{K}{s}}{\sigma\sqrt{t}}$$

Hence

$$S(t) > K \Leftrightarrow Z > \sigma\sqrt{t} - \omega$$

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4) a) Lectures:

Cost of option is $C_D = C(\tilde{s}, t, K, \sigma, r)$

Black-Scholes formula with s replaced by $\tilde{s} = s \cdot e^{-ft}$

here:

$r = 0.04 \quad t = \frac{1}{3} \quad \sigma = 0.24 \quad K = 42 \quad s = 40.4 \quad f = 0.03$

$$\Rightarrow \tilde{s} = 40.4 \cdot e^{-0.03 \cdot \frac{1}{3}} = 40.00$$

$$\omega := \frac{rt + \frac{1}{2}\sigma^2 t - \log \frac{K}{\tilde{s}}}{\sigma \sqrt{t}} = -0.1866$$

$$\omega - \sigma\sqrt{t} = -0.3252$$

Black-Scholes-formula

$$C_D = \tilde{s}\,\Phi(\omega) - Ke^{-rt}\,\Phi(\omega - \sigma\sqrt{t}) = £\,1.62$$ (30)

from tables, using $\Phi(-x) = 1 - \Phi(x)$

$$\text{prob}\,(S(t) > K) = \Phi(\omega - \sigma\sqrt{t}) = 0.373$$ (20)

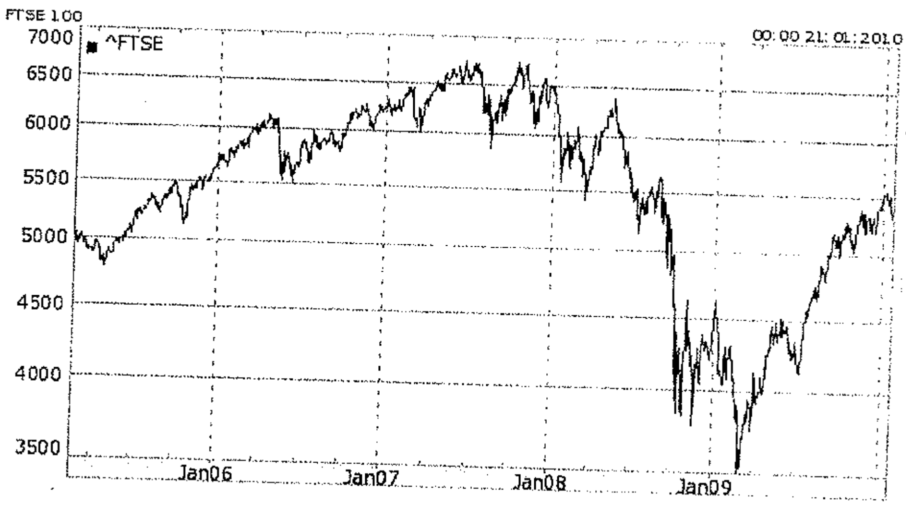
For fixed divident payment, one has to use Black-Scholes with s replaced by $\hat{s} = s \cdot (1 - f_{new})$ (lectures)

Same option cost if $\tilde{s} = \hat{s}$

$$\Leftrightarrow s e^{-ft} = s\,(1 - f_{new})$$

$$\Rightarrow f_{new} = 1 - e^{-ft} = 0.00995$$ (20)

b)



c)

lecturer's guess:

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Booming economy until summer 2007, then financial crisis unfolds ($\Rightarrow$ shares fluctuate strongly and go down), collapse of Lehman brothers Sept. 08, minimum in March 09, from then on steady increase due to recovery hopes. (30)